LECTURE 13

IV. INTERACTING FIELDS

$$\langle f | \underset{\sim}{S} | i \rangle$$

$$S = \sum_{n=0}^{\infty} \frac{(-i)^n}{n!} \int d^4x_1 \dots \int d^4x_n \, T\left(\mathcal{H}_1(x_1) \dots \mathcal{H}_1(x_n)\right)$$

$\uparrow$

$\Rightarrow$ WICK'S THEOREM

$T(\quad) = \sum \; N(\quad)$ WITH ALL POSSIBLE CONTRACTIONS

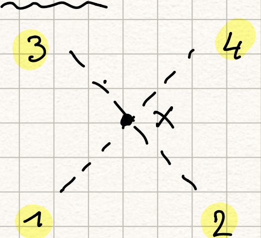
$$\underbrace{\phi(x)\,\phi(y)} = i\,\Delta_F(x-y)$$

3) FEYNMAN DIAGRAMS FOR ϕ^4

$$\mathcal{H}_1(x) = \frac{\lambda}{4!}\phi^4(x)$$

$\Rightarrow$ $\underset{\sim}{n=1}$ $\quad 2 \to 2$ SCATTERING

$|f\rangle$



$|i\rangle$

$$S^{(1)} = -\frac{i\lambda}{4!}\int d^4x \; \phi^4(x)$$

$$\langle f | S^{(1)} | i \rangle$$

$$|i\rangle = |\vec{p}_1 \, \vec{p}_2\rangle = \sqrt{2E_{p_1}}\sqrt{2E_{p_2}}\, a^\dagger(\vec{p}_1)\, a^\dagger(\vec{p}_2)|0\rangle$$

$$|f\rangle = |\vec{p}_3 \, \vec{p}_4\rangle = \sqrt{2E_{p_3}}\sqrt{2E_{p_4}}\, a^\dagger(\vec{p}_3)\, a^\dagger(\vec{p}_4)|0\rangle$$

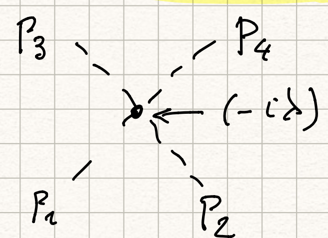
$$\langle 0|\phi(x)|\vec{p}_1\rangle = \langle 0|\int \frac{d^3k}{(2\pi)^3\sqrt{2E_k}} \left\{ a(\vec{k})e^{-ikx} + a^\dagger(\vec{k})e^{+ikx} \right\} \underbrace{|\vec{p}_1\rangle}_{\sqrt{2E_{p_1}}\, a^\dagger(p_1)|0\rangle}$$

$$[a(\vec{k}), a^\dagger(\vec{p}_1)] = (2\pi)^3 \delta^3(\vec{k}-\vec{p}_1)$$

$$\langle 0|\phi(x)|\vec{p}_1\rangle = e^{-ip_1x}$$

$$\langle \vec{p}_3|\phi(x)|0\rangle = e^{+ip_3x}$$

$$\langle f|S^{(1)}|i\rangle = -i\frac{\lambda}{4!}\,4! \underbrace{\int d^4x \; e^{-i(p_1+p_2-p_3-p_4)x}}_{(2\pi)^4\delta^4(p_1+p_2-p_3-p_4)}$$



$$= \underbrace{(2\pi)^4\delta^4(p_1+p_2-p_3-p_4)}_{\text{ENERGY-MOMENTUM CONSERVATION}} \underbrace{(-i\lambda)}_{\text{VERTEX.}}$$

$\Longrightarrow n = 2 \qquad 2 \to 2$ SCATTERING

$$S^{(2)} = \frac{1}{2!}\left(-\frac{i\lambda}{4!}\right)^2 \int d^4x \int d^4y \; T\left(\phi^4(x)\,\phi^4(y)\right)$$

$$\langle f|S^{(2)}|i\rangle \rightsquigarrow \langle \vec{p}_3\vec{p}_4|S^{(2)}|\vec{p}_1\vec{p}_2\rangle$$

$$\langle f|\, N\big(\phi(x)\phi(x)\phi(x)\phi(x)\phi(y)\phi(y)\phi(y)\phi(y)\big)|i\rangle$$

$$\phi(x)\,\phi(x)\,\phi(y)\,\phi(y)$$

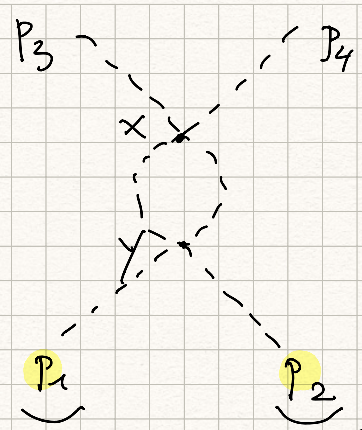


DIAGRAM (a)

$$\langle f |\, S_a^{(2)} \,| i \rangle$$

$P_1 : 4 \,, \; P_2 : 3$

$P_3 : 4 \,, \; P_4 : 3$

CONTR : 2 $\qquad x \leftrightarrow y : 2$

$$= \frac{1}{2!} \frac{(-i\lambda)}{4!} \frac{(-i\lambda)}{4!} \underbrace{(4!)^2}_{\text{SYMMETRY}} \int d^4x \int d^4y \; e^{-i(P_1+P_2)y} \cdot e^{+i(P_3+P_4)x}$$

$$i\Delta_F(x-y) \;\; i\Delta_F(x-y)$$

$$\underbrace{\phi(x)\,\phi(y)} = i\Delta_F(x-y) = i\int \frac{d^4k}{(2\pi)^4} e^{-ik.(x-y)} \tilde{\Delta}_F(k^2)$$

$$\tilde{\Delta}_F(k^2) = \frac{1}{k^2 - m^2 + i\varepsilon}$$

$$= \frac{1}{2} (-i\lambda)^2 \int \frac{d^4k}{(2\pi)^4} \, \underline{i\tilde{\Delta}_F(k^2)} \int \frac{d^4\ell}{(2\pi)^4} \, \underline{i\tilde{\Delta}_F(\ell^2)}$$

$$\int d^4x \;\; e^{+i(P_3+P_4-k-\ell).x}$$

$$\int d^4y \;\; e^{-i(P_1+P_2-k-\ell).y}$$

$$(2\pi)^4 \, \delta^4(P_3+P_4-k-\ell)$$

$$(2\pi)^4 \, \delta^4(P_1+P_2-k-\ell)$$

$$= (2\pi)^4 \, \delta^4(P_1+P_2-P_3-P_4)$$

$$\cdot \, (2\pi)^4 \, \delta^4(P_1+P_2-k-\ell)$$

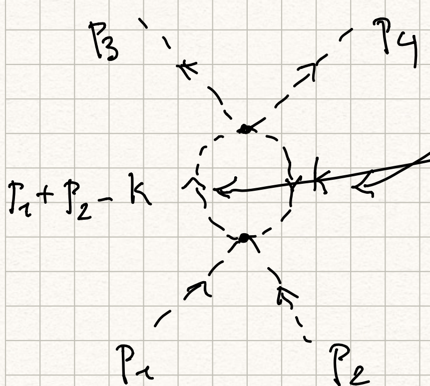
$$\Downarrow$$

$$\ell = p_1 + p_2 - k$$

$$\langle f | S_a^{(2)} | i \rangle$$

$$= \frac{1}{2} \underbrace{(-i\lambda)^2}\ \underbrace{\int \frac{d^4k}{(2\pi)^4}}\ \underbrace{i\tilde{\Delta}_F(k^2)}\ \underbrace{i\tilde{\Delta}_F\left((p_1+p_2-k)^2\right)}$$

$$\Uparrow \qquad \cdot (2\pi)^4\, \delta^4(p_1 + p_2 - p_3 - p_4)$$



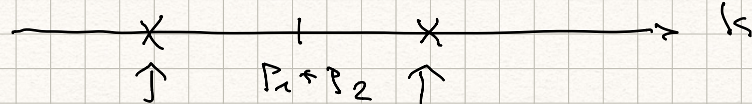
$$\overset{k}{- - \rightarrow - -} \qquad i\tilde{\Delta}_F(k^2) = \frac{i}{k^2 - m^2 + i\varepsilon}$$

2 VERTICES $\rightarrow (-i\lambda)^2$

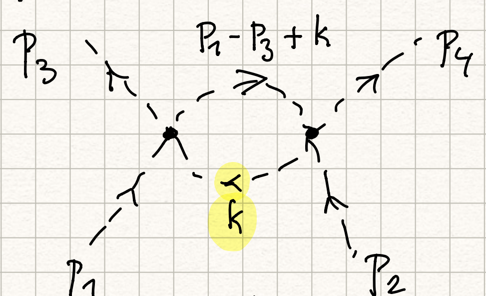
GLOBAL E-M CONSERVATION

$$(2\pi)^4\, \delta^4(p_1 + p_2 - p_3 - p_4)$$

<u>LOOP</u> $\Longrightarrow \int \frac{d^4k}{(2\pi)^4}$



$\hookrightarrow$ OTHER TOPOLOGIES



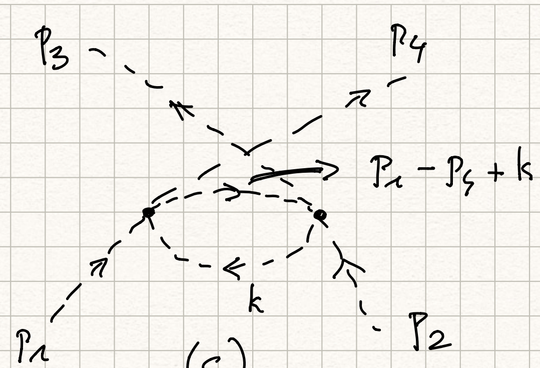
(b)

$$\langle f | S_b^{(2)} | i \rangle$$

$$= (2\pi)^4\, \delta^4(p_1 + p_2 - p_3 - p_4)$$

$$\cdot \frac{1}{2} (-i\lambda)^2 \int \frac{d^4k}{(2\pi)^4}\ i\tilde{\Delta}_F(k^2)$$

$$\cdot\, i\tilde{\Delta}_F\left((p_1 - p_3 + k)^2\right)$$

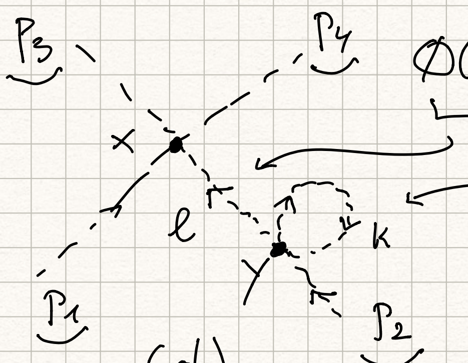


(c)

$$\langle f | S_c^{(2)} | i \rangle = \ldots \quad i\tilde{\Delta}_F\left((p_1 - p_4 + k)^2\right)$$

SPIN 0 $\quad \overset{k}{- - - - - -} \Rightarrow \dfrac{i}{k^2 - m^2 + i\varepsilon}$

↳



$$\underbrace{\phi(x)\,\phi(y)}\;\underbrace{\phi(y)\,\phi(y)}$$

(d)

$$\langle \bar{p}_3\, \bar{p}_4 | S_d^{(2)} | \bar{p}_1\, p_2 \rangle$$

$$= \frac{1}{2!}\left(\frac{-i\lambda}{\cancel{4!}}\right)^2 \underbrace{4!\,4.3.2}_{\cancel{(4!)^2}} \int d^4x \int d^4y \; e^{-ip_1 x}\, e^{+i(p_3+p_4)x} \cdot e^{-ip_2 y}\; \underbrace{i\Delta_F(x-y)}\quad \underbrace{i\Delta_F(0)}$$

$$= \frac{1}{2}(-i\lambda)^2 \int \frac{d^4k}{(2\pi)^4}\, i\tilde{\Delta}_F(k^2) \cdot \int \frac{d^4\ell}{(2\pi)^4}\, i\tilde{\Delta}_F(\ell^2)$$

$$\int d^4x \int d^4y \quad e^{-il(x-y)} \quad e^{-i(p_1-p_3-p_4)x} \, e^{-ip_2 y}$$

$$(2\pi)^4 \, \delta^4(p_2 - l)$$

$$\cdot \, (2\pi)^4 \, \delta^4(p_1 - p_3 - p_4 + \underset{\downarrow \atop p_2}{l})$$

$$\langle p_3 \, p_4 | S_d^{(2)} | p_1 \, p_2 \rangle$$

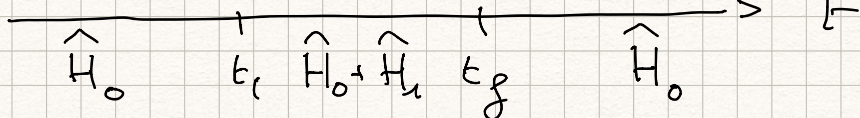
$$= (2\pi)^4 \, \delta^4(p_1 + p_2 - p_3 - p_4)$$

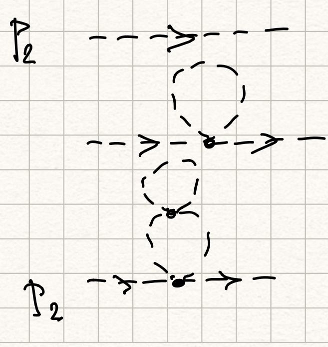
$$\cdot \frac{1}{2} (-i\lambda)^2 \; i\tilde{\Delta}_F(p_2^2) \int \frac{d^4k}{(2\pi)^4} \; i\tilde{\Delta}_F(k^2)$$

$$\uparrow$$

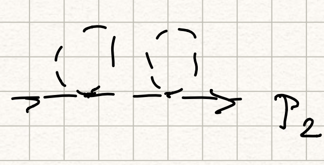
$$\frac{1}{p_2^2 - m^2 + i\varepsilon} \longrightarrow \underbrace{\frac{1}{0 + i\varepsilon}}$$

$$p_2^2 = m^2$$





$$|\vec{p}_2\rangle = \sqrt{2E_{\vec{p}_2}} \, a^+(\vec{p}_2) \, |0\rangle$$



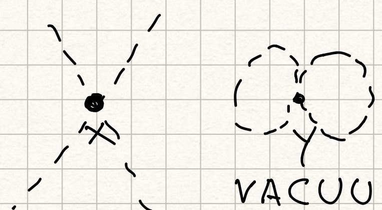
$\Rightarrow$ REDEFINITION OF $|\tilde{i}\rangle \Rightarrow |i'\rangle$

$\uparrow$

INCLUDES LOOPS ON EXT. LINES

$\hookrightarrow \quad \langle f | S^{(2)} | i \rangle$

$\langle p_3 \, p_4 | \qquad | p_1 \, p_2 \rangle$



DISCONNECTED

VACUUM BUBBLE

$$N\left(\phi(x)\phi(x)\phi(x)\phi(x)\;\phi(y)\phi(y)\phi(y)\phi(y)\right)$$

$|0\rangle \Rightarrow$ PHYSICAL VACUUM $|\Omega\rangle$ INCLUDES VACUUM BUBBLES

$$\Rightarrow \quad |\vec{p}\rangle = \sqrt{2E_{\vec{p}}}\, a^{\dagger}(p)\, |\Omega\rangle$$

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