Theoretical Physics 6a (QFT): SS 2020
Exercise sheet 10

22.06.2020

Exercise 1. (20 points) : Wick rotation

Consider the integral in D-dimensions:

$$I = \int \frac{d^D k}{(2\pi)^D} \frac{1}{(k^2 - \Delta + i\varepsilon)^2}.$$

In the lectures we performed this integral using the Wick rotation with $\Delta > 0$. Go through the same steps as in the lecture notes to show that the Wick rotation method still works for $\Delta < 0$. What is the difference between both cases?

Exercise 2. (40 points) : Dimensional regularization

Calculate the following integrals

$$\int \frac{\mathrm{d}^D k}{(2\pi)^D} \frac{k^\mu k^\nu k^\sigma}{(k^2 - \Delta + i\epsilon)^n}$$
$$\int \frac{\mathrm{d}^D k}{(2\pi)^D} \frac{k^\mu k^\nu k^\sigma k^\rho}{(k^2 - \Delta + i\epsilon)^n}$$
$$\int \frac{\mathrm{d}^D k}{(2\pi)^D} \frac{k^\mu k^\nu k^\sigma k^\rho k^\alpha}{(k^2 - \Delta + i\epsilon)^n}$$
$$\int \frac{\mathrm{d}^D k}{(2\pi)^D} \frac{k^\mu k^\nu k^\sigma k^\rho k^\alpha k^\beta}{(k^2 - \Delta + i\epsilon)^n}.$$

Exercise 3. (40 points) : Feynman parameterizations

Prove the following generalizations of the Feynman parametrization (by explicitly working out the integrals on the right hand sides):

(a)(20 points)

$$\frac{1}{A_1 A_2 A_3} = \Gamma(3) \int_0^1 dz_1 \int_0^{z_1} dz_2 \frac{1}{[A_1 + (A_2 - A_1)z_1 + (A_3 - A_2)z_2]^3}.$$

(b)(20 points)

$$\frac{1}{A^\alpha B^\beta} = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \int_0^1 dx \frac{x^{\alpha-1}(1-x)^{\beta-1}}{[B + (A-B)x]^{\alpha+\beta}}.$$