Theoretical Physics 6a (QFT): SS 2020
Exercise sheet 6

25.05.2020

Exercise 1. (50 points) : Scalar Quantum Electrodynamics

Consider the Lagrangian for the charged Klein-Gordon Field

$$\mathcal{L} = (\partial_\mu \phi)^\dagger (\partial^\mu \phi) - m^2 \phi^\dagger \phi \tag{1}$$

- Check that this Lagrangian is not invariant under U(1) gauge transformation of the form $\phi(x) \to \phi'(x) = e^{i\alpha(x)}\phi(x)$;
- Replace the derivatives by the convariant one $D_\mu = \partial_\mu + ieA_\mu$ and check whether now the Lagrangian is invariant under the local symmetry;
- Write down the interaction Lagrangian and identify the terms. Sketch each one as a different diagram;

Exercise 2. (50 points) : Gauge transformation

Consider a state $|\Psi_T\rangle$ which only contains transverse photons. Futhermore, construct a state $|\Psi'_T\rangle$ as:

$$|\Psi'_T\rangle = \left\{1 + \alpha \left[a^\dagger(\vec{k}, 3) - a^\dagger(\vec{k}, 0)\right]\right\} |\Psi_T\rangle,$$

with α a constant. Show that replacing $|\Psi_T\rangle$ by $|\Psi'_T\rangle$ corresponds to a gauge transformation:

$$\langle \Psi'_T | A^\mu(x) | \Psi'_T \rangle = \langle \Psi_T | A^\mu(x) + \partial^\mu \Lambda | \Psi_T \rangle \,,$$

where Λ is given by:

$$\Lambda(x) = \mathrm{Re}\left(i\alpha \frac{\sqrt{2}}{\omega_k^{3/2}} e^{-ik\cdot x} \right).$$