LECTURE 11

III) SPIN 1

3) FEYNMAN PROPAGATOR

- **2-POINT CORRELATOR**

$$\langle 0 | A^\mu(x) \, A^\nu(y) | 0 \rangle \equiv i D^{\mu\nu}(x-y)$$

$$A^\mu(x) = \sum_{\lambda=0}^{3} \int \frac{d^3\bar{k}}{(2\pi)^3 (2\omega_{\bar{k}})^{1/2}} \Big\{ a(\bar{k},\lambda) e^{-ikx} \varepsilon^\mu(\bar{k},\lambda) + a^\dagger(\bar{k},\lambda) e^{+ikx} \varepsilon^{\mu *}(\bar{k},\lambda) \Big\}$$

$$\omega_{\bar{k}} = |\bar{k}|$$

$$i D^{\mu\nu}(x-y) = \sum_{\lambda} \sum_{\lambda'} \int \frac{d^3\bar{k}}{(2\pi)^3 (2\omega_{\bar{k}})^{1/2}} \int \frac{d^3\bar{k}'}{(2\pi)^3 (2\omega_{\bar{k}'})^{1/2}}$$

$$\langle 0 | \Big\{ \underline{a(\bar{k},\lambda)} e^{-ikx} \varepsilon^\mu(\bar{k},\lambda) + \cancel{\cdots} \Big\} \cdot \Big\{ \cancel{\cdots} + a^\dagger(\bar{k}',\lambda') e^{+ik'y} \varepsilon^{\nu *}(\bar{k}',\lambda') \Big\} | 0 \rangle$$

$$\downarrow$$

$$\big[a(\bar{k},\lambda), \, a^\dagger(\bar{k}',\lambda') \big] = \zeta_\lambda \, \delta_{\lambda\lambda'} (2\pi)^3 \, \delta^3(\bar{k} - \bar{k}')$$

$$\zeta_0 = -1 \, , \quad \zeta_i = +1 \quad i = 1, 2, 3$$

$$i D^{\mu\nu}(x-y) = \int \frac{d^3\bar{k}}{(2\pi)^3 (2\omega_{\bar{k}})} e^{-ik\cdot(x-y)}$$

$$\sum_\lambda \zeta_\lambda \, \underbrace{\varepsilon^\mu(\vec{k},\lambda)\,\varepsilon^{\nu *}(\vec{k},\lambda)}_{\alpha\, g^{\mu\nu} + \beta \frac{k^\mu k^\nu}{k^2}}$$

LORENZ GAUGE $\quad \underline{\underline{-g^{\mu\nu}}}$

$$k^\mu\left(\omega_{\vec{k}},\ 0,\ 0,\ \omega_{\vec{k}}\right) \qquad \omega_{\vec{k}} = |\vec{k}|$$

$$\varepsilon^\mu(k,0) = (1,0,0,0)$$
$$\varepsilon^\mu(k,1) = (0,1,0,0)$$
$$\varepsilon^\mu(k,2) = (0,0,1,0)$$
$$\varepsilon^\mu(k,3) = (0,0,0,1)$$

$\mu = \nu = 0 \quad (-1) \qquad -g^{00} = -1$

$\mu = \nu = 1 \quad (+1) \qquad -g^{11} = \underline{+1}$

$$i D^{\mu\nu}(x-y) = \underset{\sim\sim\sim}{(-g^{\mu\nu})} \underbrace{\int \frac{d^3\vec{k}}{(2\pi)^3\, 2\omega_{\vec{k}}}\, e^{-ik\cdot(x-y)}}_{\Delta(x-y)} \qquad k^0 = \omega_{\vec{k}}$$

- FEYNMAN PROPAGATOR

$$\langle 0 |\, \underbrace{T\ A^\mu(x)\, A^\nu(y)}\, | 0 \rangle \equiv i D_F^{\mu\nu}(x-y)$$

$$\theta(x^0 - y^0)\ A^\mu(x)\, A^\nu(y)$$
$$+\ \theta(y^0 - x^0)\ A^\nu(y)\, A^\mu(x)$$

$$i\, D_F^{\mu\nu}(x-y) = \theta(x^0-y^0)\,(-g^{\mu\nu})\,\Delta(x-y)$$

$$+\ \theta(y^0-x^0)\,(-g^{\mu\nu})\,\Delta(y-x)$$

$$= \underbrace{(-g^{\mu\nu})}\ \underbrace{\Delta_F(x-y)}_{\hookrightarrow\ m=0}$$

↳ MOMENTUM SPACE

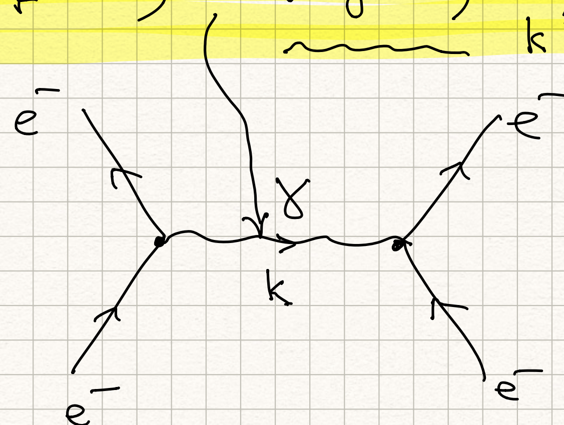
$$D_F^{\mu\nu}(x) \equiv \int \frac{d^4k}{(2\pi)^4}\, e^{-ik\cdot x}\, \tilde{D}_F^{\mu\nu}(k)$$

$$\Delta_F(k) = \frac{1}{k^2 - m^2 + i\varepsilon}$$

PHOTONS $\quad k^0 = \omega_{\vec{k}} = |\vec{k}|$

$\underline{m=0}$

$$\tilde{D}_F^{\mu\nu}(k) = (-g^{\mu\nu})\,\frac{1}{k^2+i\varepsilon}$$



$\Longrightarrow$ MORE GENERAL GAUGE

$$\mathcal{L} = \mathcal{L}_{DIRAC} - \underbrace{J^{\mu}_{em}}_{e\,\bar{\psi}(x)\gamma^{\mu}\psi(x)} A_{\mu} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

$$+ \ \mathcal{L}_{GAUGE-FIXING}$$

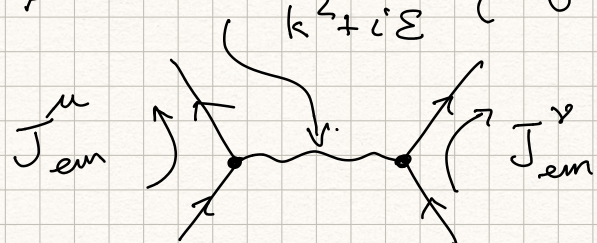
$$\mathcal{L}_{GF} = -\frac{1}{2}\left(\partial_{\mu}\hat{A}^{\mu}\right)^{2} \qquad \text{LORENZ}$$

$$= -\frac{1}{2\underbrace{\xi}}\left(\partial_{\mu}A^{\mu}\right)^{2} \qquad \text{COV. GAUGE}$$

$\xi = 1$

LORENZ | FEYNMAN

$$\tilde{D}_F^{\mu\nu}(k) = \frac{1}{k^2 + i\varepsilon}\left\{-g^{\mu\nu} + \underline{\underline{(1-\xi)}}\,\frac{k^{\mu}k^{\nu}}{k^2}\right\}$$



$k_{\mu} J^{\mu}_{em} = 0$

IV) INTERACTING FIELDS

1) S - MATRIX EXPANSION

$\Rightarrow$ TIME - EVOLUTION

- SCHRÖDINGER PICTURE

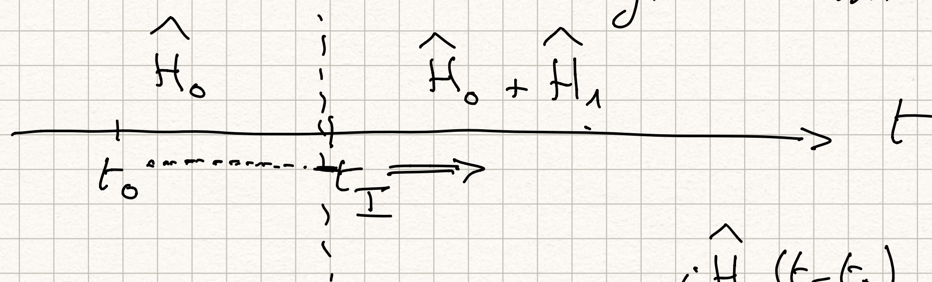
$$\hat{H}\,|\Psi(t)\rangle_S = i\frac{d}{dt}|\Psi(t)\rangle_S$$

$$|\Psi(t)\rangle = \underbrace{e^{-i\hat{H}(t-t_0)}}_{U(t,t_0)}|\Psi(t_0)\rangle_S$$

- INTERACTION PICTURE

$$\hat{H} = \hat{H}_0 + \hat{H}_1$$

$\hookrightarrow$ e.g. PERTURBATION



$\underline{t < t_I}$ $\quad |\Psi(t)\rangle_S = \underbrace{e^{-i\hat{H}_0(t-t_0)}}_{U_0}|\Psi(t_0)\rangle_S$

$\forall t$ $\quad |\Psi(t)\rangle_I \equiv U_0^{+}\,\underbrace{|\Psi(t)\rangle_S}_{U_0|\Psi(t_0)\rangle_S}$

$$U_0^{\dagger} U_0 = 1$$

$$t < t_I \qquad |\Psi(t)\rangle_I = |\Psi(t_0)\rangle_S$$

CONSTANT

- MATRIX ELEMENT

$$_S\langle \Psi(t) | \hat{O}^S | \Psi(t) \rangle_S$$

$$= {}_I\langle \Psi(t) | \hat{O}^I(t) | \Psi(t) \rangle_I$$

$$|\Psi(t)\rangle_S = \underbrace{U_0}_{e^{-i H_0 (t - t_0)}} |\Psi(t)\rangle_I$$

$$_I\langle \Psi(t) | \underbrace{U_0^{\dagger} \hat{O}^S U_0} | \Psi(t) \rangle_I$$

$$\hat{O}^I(t) \equiv U_0^{\dagger} \hat{O}^S U_0$$

$$i \frac{d}{dt} \hat{O}^I(t) = - \hat{H}_0 U_0^{\dagger} \hat{O}^S U_0 + U_0^{\dagger} \hat{O}^S U_0 \hat{H}_0$$

$$= \left[\hat{O}^I(t), \hat{H}_0 \right]$$

SPECIAL CASE $\hat{O}^I(t) = \hat{H}_0 = $ CONSTANT

• TIME DEPENDENCE OF STATES

$$i\frac{d}{dt}|\Psi(t)\rangle_I = i\frac{d}{dt}\Big(\underbrace{U_0^+}_{e^{i\hat{H}_0(t-t_0)}}|\Psi(t)\rangle_S\Big)$$

$$= -\hat{H}_0\,U_0^+\,|\Psi(t)\rangle_S$$

$$+\;U_0^+\,\underbrace{i\frac{d}{dt}|\Psi(t)\rangle_S}_{(\hat{H}_0+\hat{H}_1)|\Psi(t)\rangle_S}$$

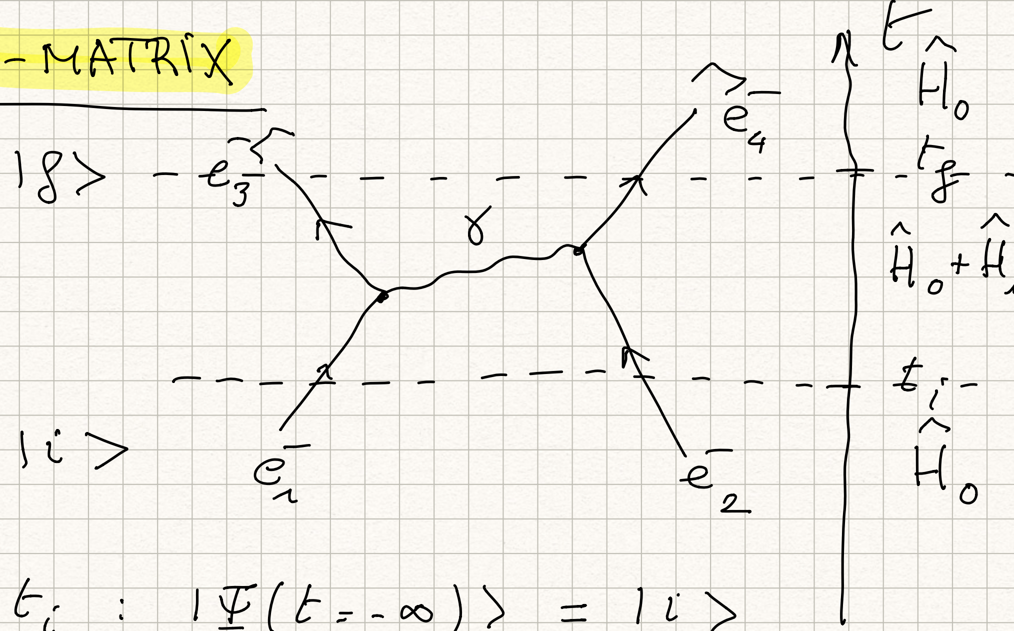
$$=\cancel{\left(-\hat{H}_0\,U_0^+ + U_0^+\,\hat{H}_0\right)}\,|\Psi(t)\rangle_S$$

$$+\;U_0^+\,\hat{H}_1\,|\Psi(t)\rangle_S$$

$$i\frac{d}{dt}|\Psi(t)\rangle_I = \underbrace{U_0^+\,\hat{H}_1\,U_0}_{\hat{H}_1^I(t)}\,|\Psi(t)\rangle_I$$

$$\Longrightarrow\quad i\frac{d}{dt}|\Psi(t)\rangle_I = \hat{H}_1^I(t)\,|\Psi(t)\rangle_I$$

⇒ S-MATRIX



$t \ll t_i$: $\underline{|\Psi(t=-\infty)\rangle_I \equiv |i\rangle}$

$$|i\rangle = |e_1^- \, e_2^- \rangle$$

DUE TO INTERACTION

$$|\Psi(t=+\infty)\rangle_I \equiv S \, |\Psi(t=-\infty)\rangle_I$$

$$S^+ S = \mathbb{1}$$

$$|f\rangle = |e_3^- \, e_4^- \rangle$$

$$\langle f \,|\Psi(t=+\infty)\rangle_I = \langle f | S | i \rangle$$

$$= S_{fi}$$

TRANSITION PROB. AMPL $i \to f$

$$\rightsquigarrow \quad \sum_f |S_{fi}|^2 = 1$$

$$S^\dagger S = \mathbb{1} \, .$$

$$\langle i | S^\dagger S | i \rangle = \langle i | i \rangle = 1$$

$$\downarrow \quad \sum_f |f\rangle\langle f|$$

$$\sum_f \underbrace{\langle i | S^\dagger | f \rangle}_{\underbrace{\langle f | S | i \rangle^*}_{S_{fi}^*}} \underbrace{\langle f | S | i \rangle}_{S_{fi}} = 1$$

$\Longrightarrow$ EXPANSION FOR S-MATRIX

$$i \frac{d}{dt} |\Psi(t)\rangle_I = \hat{H}_1^I(t) \, |\Psi(t)\rangle_I$$

$$\int_{-\infty}^{t} \quad \downarrow$$

$$i \Big(|\Psi(t)\rangle_I - \underbrace{|\Psi(t=-\infty)\rangle_I}_{|i\rangle} \Big)$$

$$= \int_{-\infty}^{t} dt_1 \; \hat{H}_1^I(t_1) \, |\Psi(t_1)\rangle_I$$

$$|\underline{\Psi}(t)\rangle_I = |i\rangle - i\int_{-\infty}^{t} dt_1\, \hat{H}_1^I(t_1)\, |\underline{\Psi}(t_1)\rangle_I$$

$$t_1 < t$$

$$
\begin{aligned}
|\underline{\Psi}(t)\rangle_I = {} & |i\rangle \\
& - i\int_{-\infty}^{t} dt_1\, \hat{H}_1^I(t_1)\, |i\rangle \\
& + (-i)^2 \int_{-\infty}^{t} dt_1\, \hat{H}_1^I(t_1) \int_{-\infty}^{t_1} dt_2\, \hat{H}_1^I(t_2)\, |i\rangle \\
& + (-i)^3 \int_{-\infty}^{t} dt_1\, \hat{H}_1^I(t_1) \int_{-\infty}^{t_1} dt_2\, \hat{H}_1^I(t_2) \\
& \qquad \cdot \int_{-\infty}^{t_2} dt_3\, \hat{H}_1^I(t_3)\, |i\rangle \\
& + \ldots
\end{aligned}
$$

$$t \to +\infty$$

$$|\underline{\Psi}(t = +\infty)\rangle \equiv S\, |i\rangle$$

$$S = \sum_{n=0}^{\infty} (-i)^n \int_{-\infty}^{+\infty} dt_1 \int_{-\infty}^{t_1} dt_2 \;\cdots \int_{-\infty}^{t_{n-1}} dt_n\; \hat{H}_1^I(t_1) \ldots \hat{H}_1^I(t_n)$$