Theoretical Physics 6a (QFT): SS 2020
Exercise sheet 1

20.04.2020

Exercise 1 (100 points): Scalar theory with $SO(2)$ invariance

Consider the following Lagrangian density of two real scalar fields $\phi_1(x)$, $\phi_2(x)$:

$$\mathcal{L} = \frac{1}{2}\left[(\partial\phi_1)^2 + (\partial\phi_2)^2\right] - \frac{m^2}{2}\left(\phi_1^2 + \phi_2^2\right) - \frac{\lambda}{4!}\left(\phi_1^2 + \phi_2^2\right)^2$$

(a)(20 points) Identify the corresponding equations of motion.

(b)(20 points) Show that the above Lagrangian is invariant under the transformations

$$\phi_1 \to \phi_1' = \phi_1 \cos\theta - \phi_2 \sin\theta,$$
$$\phi_2 \to \phi_2' = \phi_1 \sin\theta + \phi_2 \cos\theta.$$

(c)(20 points) Calculate the Noether current j_μ and show explicitly that its divergence vanishes for fields ϕ_i which satisfy equations of motion.

(d)(20 points) Show explicitly that the Noether charge Q is a conserved quantity, assuming the surface integral $\int \mathrm{d}S\, \vec{n} \cdot \vec{j}$ vanishes.

(e)(20 points) Construct the Hamiltonian density $\mathcal{H}$.