Lecture 1 (20 Apr 2020)

I Klein Gordon Field

1) Introduction & Motivation

- Special relativity

$x^\mu = (t, \vec{x})$ contra-variant vector
$x_\mu = (t, -\vec{x})$ covariant

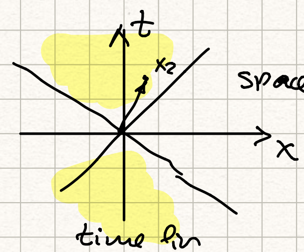
$$\left.\right\} \quad c = 1, \quad \hbar = 1$$

$$g^{\mu\nu} = g_{\mu\nu} = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}, \qquad \mu, \nu = \{0, 1, 2, 3\}$$

$$x^0 = t$$

$$x^\mu = \sum_{\nu=0}^{3} g^{\mu\nu} x_\nu = g^{\mu\nu} x_\nu \; ; \quad x_\mu = g_{\mu\nu} x^\nu$$

Space / time interval



$$x^2 \equiv x_\mu x^\mu = g_{\mu\nu} x^\nu x^\mu = t^2 - \vec{x}^2$$

Energy / momenta

$$p^\mu = (E, \vec{p}) \; ; \qquad p x \equiv p_\mu x^\mu = g_{\mu\nu} p^\nu x^\mu = E \cdot t - \vec{p}\vec{x}$$

$$p^2 \equiv p_\mu p^\mu = E^2 - \vec{p}^2 = m^2$$

$E^2 = \vec{p}^2 + m^2$ Einstein

$\underbrace{\text{Relativistic}}$ Quantum Field Theory

- Relativistic quantum mechanics

NR QM: $\quad H \Psi(\vec{x}, t) = i \dfrac{\partial}{\partial t} \Psi(\vec{x}, t)$

$$H = -\frac{1}{2m} \nabla^2 + V$$

two ways: Klein Gordon , Dirac

Klein Gordon eq.

$$\partial_\mu \equiv \frac{\partial}{\partial x^\mu} = \left(\frac{\partial}{\partial t}, \vec{\nabla}\right) \; ; \; \partial^\mu \equiv \frac{\partial}{\partial x_\mu} = \left(\frac{\partial}{\partial t}, -\vec{\nabla}\right)$$

$$\partial_\mu \partial^\mu = \frac{\partial^2}{\partial t^2} - \vec{\nabla}^2 \equiv \Box$$

(1) $$(\partial_\mu \partial^\mu + m^2)\,\varphi(x) = 0$$ spin 0 particles

$$\varphi(x) = e^{\pm ipx} \qquad px \equiv p_\mu x^\mu = Et - \vec{p}\vec{x}$$

$$\underbrace{(-p^2 + m^2)}\, e^{\pm ipx} = 0$$

$$p^2 = m^2 \qquad p^2 \equiv p_\mu p^\mu = E^2 - \vec{p}^{\,2}$$

$$E^2 = \vec{p}^{\,2} + m^2$$

Dirac eq.

$$(i\gamma^\mu \partial_\mu - m)\,\psi(x) = 0$$

$\psi(x)$ in general $(N \times 1)$ matrix $\quad \psi(x) = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \vdots \\ \psi_N \end{pmatrix}$

$$(\partial_\mu \partial^\mu + m^2)\,\psi_\alpha = 0 \qquad \alpha = 1, \dots, N$$

$$\gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu = 2 g^{\mu\nu}\, \mathbb{1}_{N\times N}$$

$\mu = 0, 1, 2, 3$ $\qquad \gamma^0, \gamma^1, \gamma^2, \gamma^3$

$\sigma^1, \sigma^2, \sigma^3$

(4x4) matrices

$N \neq 1$

$N \neq 2$

~~$N = 3$, odd~~

$N = 4$

$N = 6, 8 \dots$

Dirac matrices

$$\gamma^0 = \begin{pmatrix} 1\!\!1_{2\times 2} & 0 \\ 0 & -1\!\!1_{2\times 2} \end{pmatrix} ; \quad \vec{\gamma} = \{\gamma^1, \gamma^2, \gamma^3\}$$

$$\vec{\gamma} = \begin{pmatrix} 0 & \vec{\sigma} \\ -\vec{\sigma} & 0 \end{pmatrix}$$

$\vec{\sigma} = \{\sigma^1, \sigma^2, \sigma^3\}$ Pauli matrices

Solution of Dirac eq.

$$e^{ipx} ; \quad e^{-ipx} \qquad (i\gamma^\mu \partial_\mu - m)\Psi(x) = 0$$

$$\Psi_+(x) = N e^{-ipx} u(\vec{p}, s) ;$$

positive energy sol.

$$(\overbrace{\gamma^\mu p_\mu}^{\not{p}} - m)\, u(\vec{p}, s) = 0$$

rest frame: spin proj

$$u(\vec{p}, s) = N \begin{pmatrix} \chi_{s=\pm 1/2} \\ \frac{\vec{\sigma}\vec{p}}{E+m} \chi_{s=\pm 1/2} \end{pmatrix}$$

$$\chi_{s=+1/2} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\chi_{s=-1/2} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

covariant norm.

$$\bar{u}(\vec{p}, s)\, u(\vec{p}, s') = (2m)\, \delta_{ss'}$$

$$N = \sqrt{E+m}$$

$$\Psi_-(x) = N e^{+ipx} v(\vec{p}, s)$$

neg. energy sol

$$(\not{p} + m)\, v(\vec{p}, s) = 0 \qquad v(\vec{p}, s) = N \begin{pmatrix} \frac{\vec{\sigma}\vec{p}}{E+m} \chi'_{s=\pm 1/2} \\ \chi'_{s=\pm 1/2} \end{pmatrix}$$

absence of neg. energy sol

$\Rightarrow$ interp. as pos. energy sol for antipart.

$$\chi'_{s=+1/2} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} ; \quad \chi'_{s=-1/2} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$e^-;\ e^+$ Rel. quantum mech.

$E > (2m) \Rightarrow e^-e^+$ can be created

$\boxed{\text{Rel Quantum field theory}}$

- Particle creation / annihilation

classical field theory

K.G. field $\rightarrow$ $\varphi(x) = \sum_{\bar p} \left(a(\bar p)\, e^{-ipx} + \underbrace{b^*(\bar p)}_{\to a^*}\, e^{ipx} \right)$

$\varphi(x)$ real $\Rightarrow \varphi^* = \varphi \Rightarrow b = a$

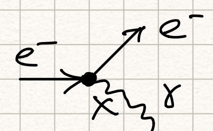
$p^2 = m^2$

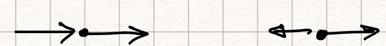
Quantization

field operator $\rightarrow$ $\underline{\hat\varphi(x)} = \sum_{\bar p} \left(\hat a(\underline{\bar p})\, e^{-ipx} + \hat a^+(\underline{\bar p})\, e^{ipx} \right)$

$\hat a(\bar p)$ annihilate particle with mom $\vec p$

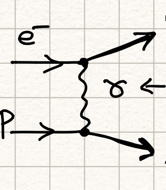
$\hat a^+(\bar p)$ create — " — $\vec p$





Relativistic QFTs

(1) E.M. quantum electrodynamic.



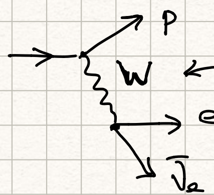
$\gamma \leftarrow$ messenger particle of this interaction

$e^\pm$, spin $1/2$

interaction = exchange of "force carrying" particles

$m_\gamma = 0$, long range interaction

(2) Weak interaction $\qquad n \to p\, e^- \bar{\nu}_e$



$\leftarrow$ messenger particles $\qquad W^\pm, Z^0 \sim 80-90$ GeV

short range inter.

(3) Strong interaction (QCD)

Quarks, spin $1/2$ $\qquad \begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix} \begin{pmatrix} t \\ b \end{pmatrix} \begin{matrix} 2/3\, e \\ -1/3\, e \end{matrix}$ $\qquad q$

$m_{u,d} \ll m_{c,s} \ll m_{t,b}$

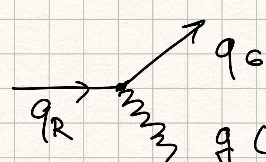
$|p\rangle = |uud\rangle \qquad +1e = (2/3 + 2/3 - 1/3)e$

$|uuu\rangle \qquad m_{u,d} \sim 5-7$ MeV

$m_p = \underbrace{938 \text{ MeV}} \neq \underbrace{2m_u + m_d}_{17 \text{ MeV}}$

$|p\rangle = c_1 |uud\rangle + c_2 |uud\, u\bar{u}\rangle + \ldots$

3 COLOR : (q_R, q_G, q_B)



g (gluon) $R\bar{G}$

$m_g = 0$

$R\bar{G}, R\bar{B}$
$G\bar{R}, G\bar{B}$
$B\bar{G}, B\bar{R}$
$R\bar{R}, G\bar{G}, B\bar{B}$
$R\bar{R} - G\bar{G}$
$R\bar{R} + G\bar{G} - 2B\bar{B}$
~~$R\bar{R} + G\bar{G} + B\bar{B}$~~

8 gluons

(2)(3) $\Rightarrow$ elem. part. physics course

(1) $\Rightarrow$ our course

4) Gravitational $\underline{\underline{\text{spin} = 2}}$

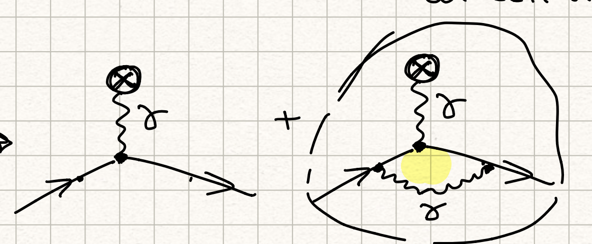
Quantum corrections

magnetic moment of the electron $\longrightarrow$ $\vec{\mu} = \frac{e}{2m} g \vec{S}$

classical $g = 1$

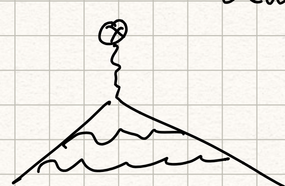
Dirac theor $g = 2$

correction



$g = 2(1 + \underset{\text{correction}}{\underset{\sim}{a}})$

$\alpha_{em} = \frac{e^2}{4\pi} = \frac{1}{137}$



$g = 2,\underbrace{\_\_\_\_}_{\text{10 digits}}0$ exp

$g = 2,\underbrace{\_\_\_\_}0$ th