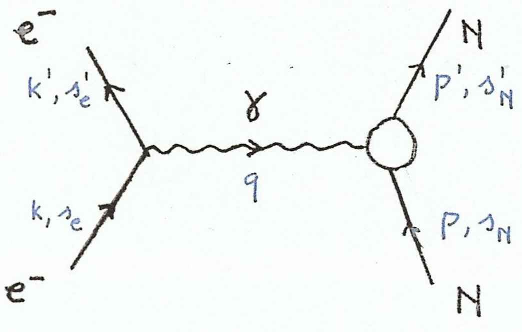
(e, e')



LAB

$k\,(\omega, \bar{k})$

$k'(\omega', \bar{k}')$

$q\,(\nu, \bar{q}) \Rightarrow Q^2 \equiv -q^2 = \bar{q}^2 - \nu^2$

$p\,(M_N, 0)$

$p'(E', \bar{p}')$

s : POLARIZATIONS

REMARKS

→ UNPOLARIZED e^- BEAM $\Rightarrow \frac{1}{2}\sum_{s_e=\pm\frac{1}{2}}$

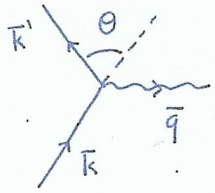
→ UNPOLARIZED NUCLEON $\Rightarrow \frac{1}{2}\sum_{s_N=\pm\frac{1}{2}}$

→ NO POLARIZATION DETECTED $\Rightarrow \sum_{s'_e}$ & $\sum_{s'_N}$

→ ONE PHOTON EXCHANGE APPROXIMATION (Z^0 EXCHANGE IS NEGLECTED)

→ $m_{e^-} \approx 0$: ULTRA - RELATIVISTIC e^- $\quad U(\bar{k}, s) = \sqrt{\omega}\begin{pmatrix} \chi_s \\ \frac{\bar{\sigma}\cdot\bar{k}}{\omega}\chi_s \end{pmatrix}$

$\omega = |\bar{k}|$

→ 

$Q^2 = 4\omega\omega' \sin^2\frac{\theta}{2}$

* CROSS SECTION IN LAB SYSTEM

$$\boxed{\begin{aligned} d\sigma^{LAB} = &\left(\frac{1}{\underset{\substack{\parallel \\ 1}}{(v_{rel})}\,(2\omega)(2M_N)}\right)\left(\frac{d^3\bar{k}'}{(2\pi)^3\, 2\omega'}\right)\left(\frac{d^3\bar{p}'}{(2\pi)^3\, 2E'}\right)(2\pi)^4\,\delta^4(k+p-k'-p') \\ &\frac{1}{2}\sum_{s_e}\sum_{s_e'}\ \frac{1}{2}\sum_{s_N}\sum_{s_N'}\Big|\,\bar{u}(\bar{k}', s_e')\,\gamma_\mu\, u(\bar{k}, s_e)\,\frac{e^2}{Q^2} \\ &\qquad \langle p', s'\,|\,\hat{J}^\mu_{EM}(0)\,|\,p, s\rangle\Big|^2 \end{aligned}}$$

(I.1)

$\underline{\underline{\hat{J}^{EM}_\mu(0)}}$ IS NUCLEON EM CURRENT OPERATOR (IN 2° QUANTIZATION) IN COORDINATE SPACE FOR $x = 0$

OUTGOING NUCLEON IS NOT OBSERVED $\Rightarrow$ ITS PHASE SPACE IS INTEGRATED OVE

$$\int \frac{d^3\bar{p}'}{(2\pi)^3\, 2E'}\ (2\pi)^4\,\delta^4(k+p-k'-p')\ \ldots$$

$$= (2\pi)\ \delta\left(p'^2 - M_N^2\right)$$

$$\downarrow \quad p' = p + q$$

$$= \frac{(2\pi)}{2M_N}\ \delta\left(\nu - \frac{Q^2}{2M_N}\right)$$

$$= \frac{2\pi}{2M_N}\ \delta\left(\omega - \omega' - \frac{4\omega\omega'}{2M_N}\sin^2\theta/2\right)$$

$$= \frac{2\pi}{2M_N}\left(\frac{\omega'}{\omega}\right)\ \delta\left(\omega' - \frac{\omega}{\left[1 + \frac{4\omega}{2M_N}\sin^2\theta/2\right]}\right)$$

" "

ELASTIC SCATTERING

$$Q^2 = 2M_N \nu$$

$$\downarrow$$

$$\bar{q}^2 = \nu^2 + 2M_N \nu$$

FOR A GIVEN MOMENTUM TRANSFER $\bar{q}$,

THE ENERGY TRANSFER ν IS FIXED

$\downarrow$

ω' IS NOT AN INDEPENDENT VARIABLE FOR ELASTIC SCATTERING

$\to$ ELASTIC

$$\left(\frac{d\sigma}{d\Omega_{k'}}\right)^{EL,\,LAB} = \int d\omega' \left(\frac{d\sigma}{d\Omega_{k'}\, d\omega'}\right)^{LAB}$$

$\downarrow$ FROM (I.1) IT FOLLOWS

$$\boxed{\left(\frac{d\sigma}{d\Omega_{k'}}\right)^{EL,\,LAB} = \frac{\alpha^2}{(2M_N)^2 Q^4} \left(\frac{\omega'}{\omega}\right)^2 L_{\mu\nu} H^{\mu\nu} \Bigg|_{\frac{\omega'}{\omega} = 1 - \frac{Q^2}{2M\omega}}} \qquad (I.2)$$

WITH

$$\alpha = \frac{e^2}{4\pi} \simeq \frac{1}{137} \qquad (I.3)$$

→ LEPTON TENSOR $L_{\mu\nu}$

$$\boxed{L_{\mu\nu} \equiv \frac{1}{2} \sum_{s_e} \sum_{s_e'} \bar{u}(k', s_e')\, \gamma_\mu\, u(k, s_e)\, \bar{u}(k, s_e)\, \gamma_\nu\, u(k', s_e')} \qquad (I.4)$$

→ HADRONIC TENSOR $H^{\mu\nu}$

$$\boxed{H^{\mu\nu} \equiv \frac{1}{2} \sum_{s_N} \sum_{s_N'} \langle p' s' |\, \hat{J}^{\mu}_{EM}(0)\, | p, s \rangle \langle p, s |\, \hat{J}^{\nu\dagger}_{EM}(0)\, | p', s' \rangle} \qquad (I.5)$$

* $\boxed{\text{LEPTON TENSOR}}$ $L_{\mu\nu}$

$$L_{\mu\nu} = \frac{1}{2} \sum_{s_e} \sum_{s_e'} \bar{u}(k', s_e') \gamma_\mu u(k, s_e) \bar{u}(k, s_e) \gamma_\nu u(k', s_e')$$

$$\Big\downarrow \quad \sum_{s_e} u(k, s_e) \bar{u}(k, s_e) = \not{k}$$

$$= \frac{1}{2} \, Tr \left[\gamma_\mu \not{k} \gamma_\nu \not{k}' \right]$$

$$= 2 \left[k_\mu k'_\nu + k_\nu k'_\mu - (k.k') g_{\mu\nu} \right]$$

$$\Big\downarrow \quad Q^2 = 2 k.k'$$

$$\boxed{L_{\mu\nu} = 2 k_\mu k'_\nu + 2 k_\nu k'_\mu - Q^2 g_{\mu\nu}} \qquad (I.6)$$

REMARK : $\quad q^\mu L_{\mu\nu} = q^\nu L_{\mu\nu} = 0$

* $\boxed{\text{HADRONIC TENSOR}}$ $H^{\mu\nu}$

→ NUCLEON ELECTROMAGNETIC CURRENT :

$$\boxed{\begin{aligned}&\langle p', s'_N \,|\, \hat{J}^{\mu}_{EM}(0) \,|\, p, s_N \rangle \\ &= \bar{N}(p', s'_N) \left\{ F_1^N(Q^2)\, \gamma^{\mu} + F_2^N(Q^2)\, i\sigma^{\mu\nu} \frac{q_\nu}{2M_N} \right\} N(p, s)\end{aligned}}$$

(I.7)

↓ GORDON IDENTITY (FOR ON-SHELL NUCLEONS)

$$= \bar{N}(\bar{p}', s'_N) \left\{ \underbrace{F_1^N(Q^2) \frac{(p+p')^{\mu}}{2M_N}}_{\text{CONVECTION CURRENT}} + \underbrace{\left[F_1^N(Q^2) + F_2^N(Q^2)\right] \frac{i\sigma^{\mu\nu} q_\nu}{2M_N}}_{\text{MAGNETIZATION CURRENT}} \right\} N(\bar{p}$$

* $F_1^N(Q^2)$: DIRAC FORM FACTOR FOR NUCLEON N

$F_2^N(Q^2)$: PAULI FORM FACTOR " " "

* SACHS FORM FACTORS (MAGNETIC (M) , ELECTRIC (E))

$$\boxed{\begin{aligned} G_M^N &\equiv F_1^N + F_2^N \\ G_E^N &\equiv F_1^N - \frac{Q^2}{4M_N^2} F_2^N \end{aligned}} \Rightarrow \boxed{\begin{aligned} F_1^N(Q^2) &= \frac{\frac{Q^2}{4M_N^2} G_M^N + G_E^N}{1 + Q^2/4M_N^2} \\ F_2^N(Q^2) &= \frac{G_M^N - G_E^N}{1 + Q^2/4M_N^2} \end{aligned}}$$

(I.8)

* EXPERIMENTAL INFORMATION ABOUT NUCLEON ELECTROMAGNETIC FORM FACTORS

e.g.

↳ GALSTER et al., NUCL. PHYS. B32 (1971) 221

↳ PLATCHKOV et al., NUCL. PHYS. A510 (1990) 740

PROTON

$$\begin{cases} F_1^p(Q^2=0) = 1 \\ F_2^p(Q^2=0) = 1{,}79 \end{cases} \qquad \begin{cases} G_E^p(Q^2=0) = 1 \\ G_M^p(Q^2=0) = 2{,}79 = \mu_p \end{cases}$$

NEUTRON

$$\begin{cases} F_1^n(Q^2=0) = 0 \\ F_2^n(Q^2=0) = -1{,}91 \end{cases} \qquad \begin{cases} G_E^n(Q^2=0) = 0 \\ G_M^n(Q^2=0) = -1{,}91 = \mu_n \end{cases}$$

$$\boxed{G_E^p(Q^2) \approx \frac{G_M^p(Q^2)}{\mu_p} \approx \frac{G_M^n(Q^2)}{\mu_n} = \underset{\text{DIPOLE}}{G_D(Q^2)}} \qquad (I.9)$$

$$\boxed{G_D(Q^2) = \frac{1}{\left(1 + \frac{Q^2}{M_V^2}\right)^2}, \quad M_V \simeq 843 \text{ MeV}} \qquad (I.10)$$

$$\boxed{G_E^n(Q^2) \simeq -\mu_n \frac{Q^2}{4M_N^2} \frac{G_D(Q^2)}{\left(1 + \frac{Q^2}{\Lambda^2}\right)}, \quad \Lambda \simeq 794 \text{ MeV}} \qquad (I.11)$$

→ USING (I.7), THE HADRONIC TENSOR (I.5) BECOMES

$$H^{\mu\nu} = \frac{1}{2} \mathrm{Tr} \left\{ (\not p + M_N)\, \Gamma^\mu\, (\not p' + M_N)\, \Gamma^\nu \right\}$$

WITH $\Gamma^\mu = (F_1 + F_2)\,\gamma^\mu - F_2 \dfrac{(p+p')^\mu}{2M_N}$

↓

$$\begin{aligned} H^{\mu\nu} = {} & \frac{1}{2} (F_1 + F_2)^2 \,\mathrm{Tr} \left\{ (\not p + M_N)\, \gamma^\mu\, (\not p' + M_N)\, \gamma^\nu \right\} \\ & - \frac{1}{2} (F_1 + F_2) F_2 \frac{(p+p')^\mu}{2M_N} \mathrm{Tr} \left\{ (\not p + M_N)(\not p' + M_N)\, \gamma^\nu \right\} \\ & - \frac{1}{2} (F_1 + F_2) F_2 \frac{(p+p')^\nu}{2M_N} \mathrm{Tr} \left\{ (\not p + M_N)\, \gamma^\mu\, (\not p' + M_N) \right\} \\ & + \frac{1}{2} F_2^2 \frac{(p+p')^\mu}{2M_N} \frac{(p+p')^\nu}{2M_N} \mathrm{Tr} \left\{ (\not p + M_N)(\not p' + M_N) \right\} \end{aligned}$$

↓ WORKING THIS OUT USING THE STANDARD Tr RULES AND USING (I.8)

$$\boxed{\begin{aligned} H^{\mu\nu} = {} & \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2} \right) Q^2 (G_M^N)^2 \\ & + \left(p^\mu - \frac{p.q}{q^2} q^\mu \right)\left(p^\nu - \frac{p.q}{q^2} q^\nu \right) 4\, \frac{(G_E^N)^2 + \frac{Q^2}{4M_N^2} (G_M^N)^2}{1 + \frac{Q^2}{4M_N^2}} \end{aligned}}$$

(I.12)

REMARK : $q_\mu H^{\mu\nu} = q_\nu H^{\mu\nu} = 0$

MAKING THE CONTRACTION IN (I.2)

OF THE LEPTONIC TENSOR (I.6)

WITH THE HADRONIC TENSOR (I.12),

WE OBTAIN THE ELASTIC (e,e') CROSS SECTION IN LAB SYSTEM:

$$\left(\frac{d\sigma}{d\Omega_{k'}}\right)^{EL,\,LAB} = \left(\frac{d\sigma}{d\Omega_{k'}}\right)_{MOTT} \overbrace{\left(1 - \frac{Q^2}{2M_N\omega}\right)}^{\omega'/\omega} \cdot \left\{ \frac{(G_E^N)^2 + \frac{Q^2}{4M_N^2}(G_M^N)^2}{1 + \frac{Q^2}{4M_N^2}} + \frac{Q^2}{2M_N^2}\, tg^2\frac{\theta}{2}\, (G_M^N)^2 \right\} \qquad (I.13)$$

↳ THIS IS THE WELL KNOWN ROSENBLUTH FORMULA

↳ THE MOTT CROSS SECTION (WHICH DESCRIBES SCATTERING OF e^- FROM A SPIN 0 TARGET)

IS GIVEN BY

$$\boxed{\left(\frac{d\sigma}{d\Omega_{k'}}\right)_{MOTT} = \frac{\alpha^2 \cos^2\theta/2}{4\omega^2 \sin^4\theta/2}} \qquad (I.14)$$

↳ THE FACTOR $\left(1 - \frac{Q^2}{2M_N\omega}\right)$ IN (I.13) IS DUE TO RECOIL