Exercise sheet 12
Theoretical Physics 5 : WS 2019/2020
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Electron-nucleon elastic scattering

This exercise is based on "Cross section electron scattering" lecture notes.

a) (*50 p.*) Consider the hadronic tensor

$$H_{\mu\nu} = \frac{1}{2} \sum_{s_N, s'_N} \langle p', s'_N | \hat{J}_\mu(0) | p, s_N \rangle \langle p, s_N | \hat{J}^\dagger_\nu(0) | p', s'_N \rangle,$$

with nucleon electromagnetic current

$$\langle p', s'_N | \hat{J}^\mu(0) | p, s_N \rangle = \bar{N}(p', s'_N) \left[F_1^N(Q^2)\gamma^\mu + F_2^N(Q^2) i\sigma^{\mu\nu} \frac{q_\nu}{2M_N} \right] N(p, s_N),$$

where $F_{1,2}^N(Q^2)$ are Dirac and Pauli form factors for nucleon N.
Show that the hadronic tensor becomes

$$H^{\mu\nu} = \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2} \right) Q^2 \left(G_M^N\right)^2 + \left(p^\mu - \frac{p.q}{q^2} q^\mu \right) \left(p^\nu - \frac{p.q}{q^2} q^\nu \right) 4 \frac{\left(G_E^N\right)^2 + \frac{Q^2}{4M_N^2}\left(G_M^N\right)^2}{1 + \frac{Q^2}{4M_N^2}},$$

where $G_M^N = F_1^N + F_2^N$ and $G_E^N = F_1^N - \frac{Q^2}{4M_N} F_2^N$ are so-called electric and magnetic form factor.

b) (*5 p.*) Prove that $q_\mu H^{\mu\nu} = q_\nu H^{\mu\nu} = 0$.

c) (*10 p.*) Explain why the most general interaction vertex is given by

$$\Gamma_\mu = F_1^N(Q^2)\gamma^\mu + F_2^N(Q^2) i\sigma^{\mu\nu} \frac{q_\nu}{2M_N}.$$

d) (*35 p.*) Show that the elastic electron-nucleon cross section is described by Rosenbluth formula

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)^{\text{El,lab}} = \left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)^{\text{Mott}} \left(1 - \frac{Q^2}{2M_N\omega}\right) \left[\frac{\left(G_E^N\right)^2 + \frac{Q^2}{4M_N^2}\left(G_M^N\right)^2}{1 + \frac{Q^2}{4M_N^2}} + \frac{Q^2}{2M_N^2} \tan^2\frac{\theta}{2} \left(G_M^N\right)^2 \right],$$

where the Mott cross section is given by

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)^{\text{Mott}} = \frac{\alpha^2 \cos^2\theta/2}{4\omega^2 \sin^4\theta/2}.$$