Exercise sheet 13
Theoretical Physics 5 : WS 2019/2020
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Exercise 1. (50 points) :
Path integral in terms of effective action

In Solid State physics, the non-relativistic motion of electrons is often considered perturbed by the effect of impurities, as it happens in typical semiconductors. In simple cases, within one-electron approximation, one can approximate this effect with a position-dependent effective mass Schrodinger equation.

Consider a one-dimensional system with position-dependent mass:

$$H = \frac{p^2}{2f(x)}.$$

a) (*5 p.*) Show that the corresponding Lagrangian is $L = \frac{1}{2}f(x)\dot{x}^2$.

b) (*25 p.*) Write the transition amplitude in discrete space and integrate it over momentum variables.

c) (*15 p.*) Take the limit $\Delta t \to 0$, $\Delta x = x_{i+1} - x_i \to 0$.
Hint: $\frac{1}{\Delta t}\delta_{ij} \to \delta(t_i - t_j)$.

d) (*5 p.*) Show that

$$K(x_f, t_f; x_i, t_i) = \int \mathcal{D}x(t) \exp\left[\frac{i}{\hbar}S_{\text{eff}}\right],$$

with

$$S_{\text{eff}} = \int \mathrm{d}t \left[L - \frac{i\hbar}{2}\delta(0)\ln f(x)\right].$$

Exercise 2. (50 points)

Use the expression for the transition amplitude $K_0(x, t; 0, 0)$ for a free particle in one dimension as derived in the lecture. Show that K_0 satisfies a Schroedinger equation:

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}K_0(x, t; 0, 0) = i\hbar\frac{\partial}{\partial t}K_0(x, t; 0, 0).$$