Exercise sheet 11
Theoretical Physics 5 : WS 2019/2020
Lecturers : Prof. M. Vanderhaeghen, Dr. I. Danilkin

06.01.2020

Exercise 0.

How much time did it take to complete the task?

Exercise 1. (50 points)
e^+e^- annihilation into a scalar quark-antiquark pair

Consider electron-positron annihilation into a quark-antiquark pair $e^+(k_2)e^-(k_1) \longrightarrow q(p_2)\bar{q}(p_1)$. Treat the electron as a massless Dirac particle and the quark as a massless Klein-Gordon particle.

a) (*15 p.*) The squared matrix element obtained as an average over electron and positron spin configurations can be expressed as

$$|\mathcal{M}|^2 = \frac{e^4 e_q^2}{s^2} L^{\mu\nu} Q_{\mu\nu}$$

with the quark charge $e_q e$.
Find the expressions for the quark tensor $Q_{\mu\nu}$ and the leptonic tensor $L_{\mu\nu}$ in terms of electron and quark momenta.
Hint: The leptonic tensor can be expressed as $L^{\mu\nu} = \frac{1}{2} Tr[\not{k}' \gamma^\mu \not{k} \gamma^\nu]$.

b) (*15 p.*) Calculate $L^{\mu\nu}Q_{\mu\nu}$ in terms of the Mandelstam variable s and the angle between the initial electron momenta and the final anti-quark momenta in the center-of- mass frame.

c) (*10 p.*) Express the result for $L^{\mu\nu}Q_{\mu\nu}$ in terms of the Mandelstam variable $s = (k_1 + k_2)^2$ and variable $t = (k_1 - p_1)^2$.

d) (*10 p.*) Find the result for the differential cross section in the center of mass frame.

Exercise 2. (50 points) π^+e^- elastic scattering

Consider elastic unpolarized π^+e^- scattering in Quantum Electrodynamics. Treat the electron as a massless Dirac particle and the pion as a massive Klein-Gordon particle with mass m.

a) (*10 p.*) Write down the expression for the differential cross-section in the laboratory frame in terms of the matrix element.

b) (*15 p.*) Integrate over the pion phase space and electron momentum and obtain the expression for angular differential cross section in terms of the electron scattering angle and the matrix element.

c) (*15 p.*) The squared matrix element obtained as an average over initial spin configurations and sum over final electron spin configurations can be expressed as

$$|M|^2 = \frac{e^4}{t^2} L^{\mu\nu} H_{\mu\nu}.$$

Find the expressions for the hadronic tensor $H_{\mu\nu}$ and the leptonic tensor $L_{\mu\nu}$ in terms of electron and pion momenta.

d) (*10 p.*)Find the result for the differential cross section in the laboratory frame.