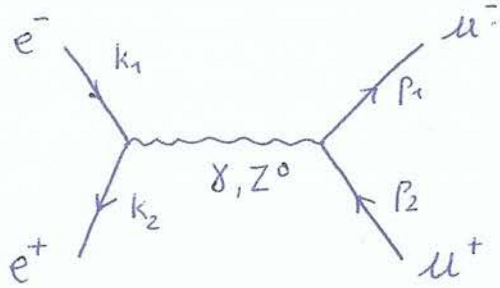
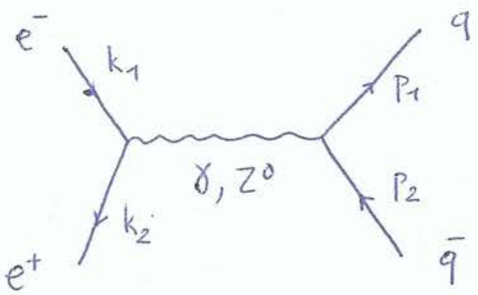
QUARK INTERACTIONS
$e^+e^- \to$ HADRONS

* $e^+e^- \to \mu^+\mu^-$ & $e^+e^- \to q\bar{q}$ ANNIHILATION



$e^-e^+ \to \mu^-\mu^+$



$e^-e^+ \to q\bar{q}$

CONSIDER e^-e^+ CM SYSTEM (cf. e^+e^- COLLIDERS)
AT HIGH C.M ENERGY $\sqrt{s}$

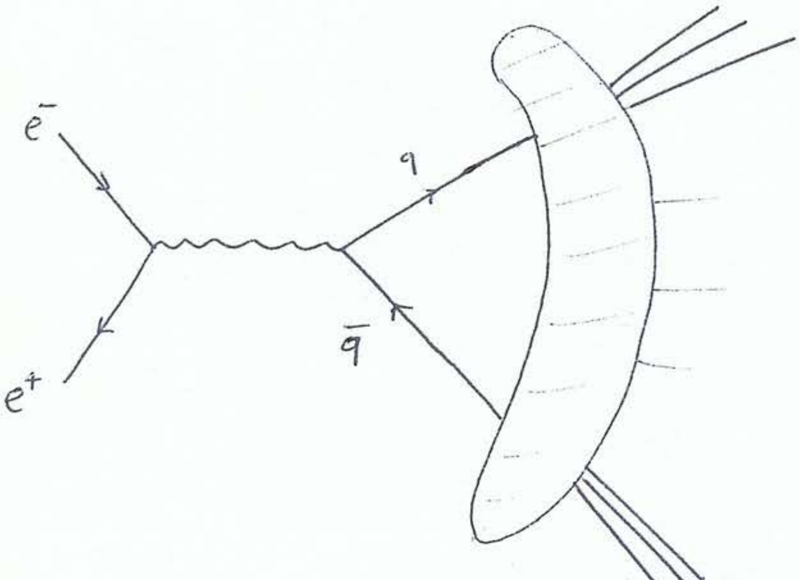
$$s := (k_1 + k_2)^2$$

LEP-II : $\sqrt{s} = 200$ GeV

↳ PRODUCTION OF QUARKS : SHORT DISTANCE PROCESS :
INVOLVES TIME / LENGTH SCALES $\sim \frac{1}{\sqrt{s}}$

e.g. $\sqrt{s} = 200$ GeV $\longleftrightarrow$ $\frac{1}{\sqrt{s}} \simeq 10^{-3}$ fm.

↳ QUARKS HADRONIZE ON THEIR WAY OUT
THIS HAPPENS AT MUCH LONGER LENGTH SCALES ~ 1 fm



HADRONIZATION

IN THIS CASE:
HADRONS WHICH COME OUT
FOR A 'SPRAY' OF COLLINEAR HADRONS : JET

↳ TOTAL MOMENTUM
↳ FLAVOR QUANTUM NUMBERS

CORRESPOND WITH THAT OF
FRAGMENTING QUARK

⇓

JET'S CAN BE SEEN AS "FOOTPRINT" OF QUARK

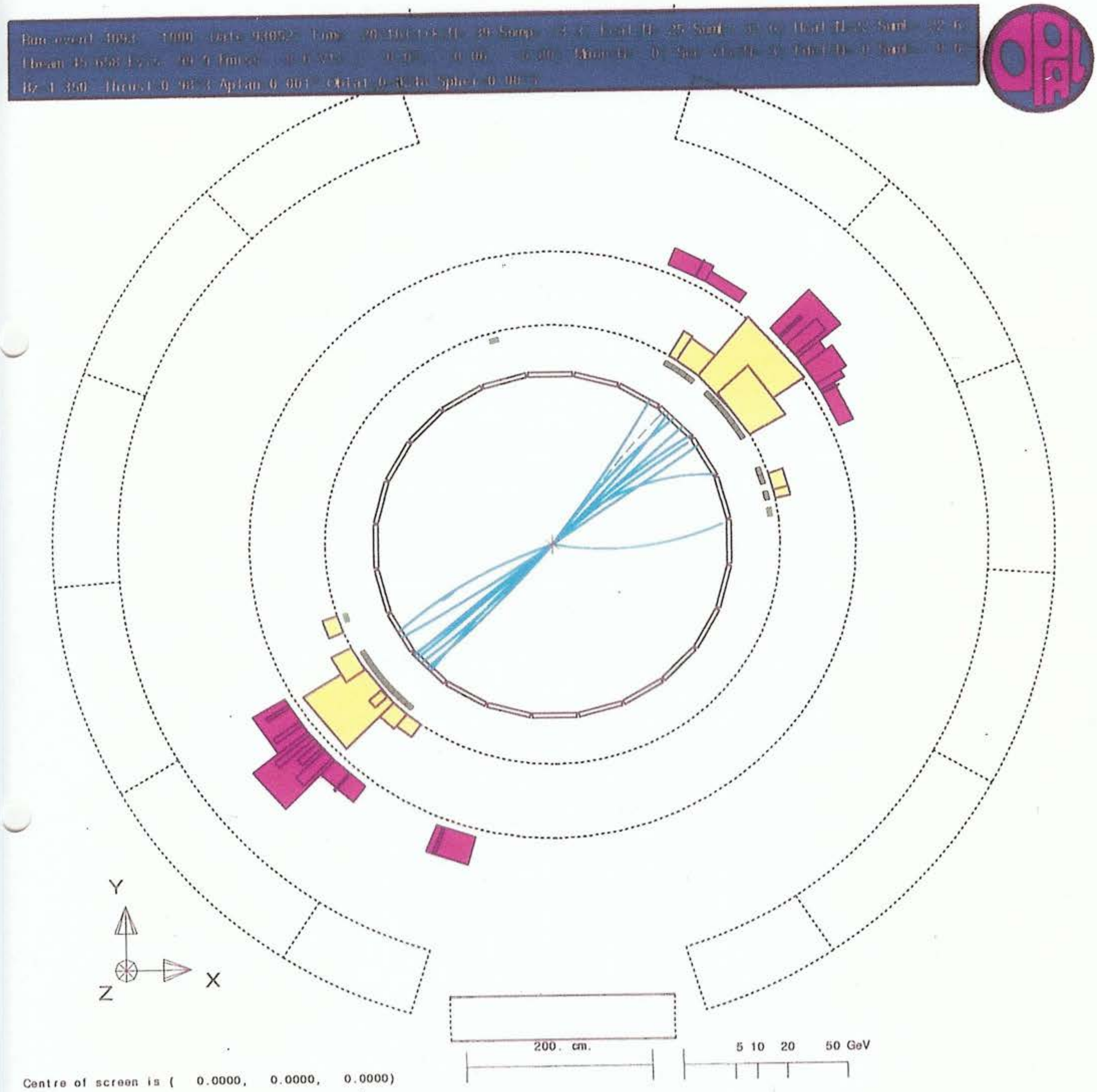
⟹ TOTAL INCLUSIVE CROSS SECTION

≡ Σ OVER ALL PARTONIC SUBPROCESSES.

$$\sigma(e^+e^- \to \text{HADRONS}) = \sum_q \sigma(e^+e^- \to q\bar{q},\ q\bar{q}g, \cdots)$$

↑ 2 JETS ↑ 3 JETS

$e^+e^- \rightarrow q\bar{q}$ (2 JET EVENT)



SOME DIRACOLOGY

- SPINOR

$$U(p,s) = \sqrt{E+M} \begin{pmatrix} \chi_s \\ \frac{\bar{\sigma}\cdot\bar{p}}{E+M}\chi_s \end{pmatrix}$$

$$\bar{U}U = 2M$$

- ENERGY PROJECTOR

$$\boxed{\sum_s U(p,s)\,\bar{U}(p\,s) = \not{p} + M}$$

PROOF

$$U(p\,s)\;\bar{U}(p\,s) = (E+M)\begin{pmatrix} \chi_s \\ \frac{\bar{\sigma}\cdot\bar{p}}{E+M}\chi_s \end{pmatrix}\begin{pmatrix} \chi_s^+ & -\chi_s^+\frac{\bar{\sigma}\cdot\bar{p}}{E+M} \end{pmatrix}$$

$$= (E+M)\begin{pmatrix} \chi_s\chi_s^+ & -\chi_s\chi_s^+\frac{\bar{\sigma}\cdot\bar{p}}{E+M} \\ \frac{\bar{\sigma}\cdot\bar{p}}{E+M}\chi_s\chi_s^+ & -\frac{\bar{\sigma}\cdot\bar{p}}{E+M}\chi_s\chi_s^+\frac{\bar{\sigma}\cdot\bar{p}}{E+M} \end{pmatrix}$$

$$\downarrow \quad \sum_s \chi_s\chi_s^+ = \mathbb{1}_{2\times2}$$

$$\sum_s U(p\,s)\;\bar{U}(p\,s) = (E+M)\begin{pmatrix} \mathbb{1}_{2\times2} & -\frac{\bar{\sigma}\cdot\bar{p}}{E+M} \\ \frac{\bar{\sigma}\cdot\bar{p}}{E+M} & -\frac{\bar{p}^2}{(E+M)^2} \end{pmatrix}$$

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$$\sum_s u(p\,s)\,\bar{u}(p\,s) = \begin{pmatrix} E+M & -\bar{\sigma}.\bar{p} \\ \bar{\sigma}.\bar{p} & -(E-M) \end{pmatrix}$$

$$= \gamma^0 . E - \bar{\gamma}.\bar{p} + M$$

$$\overset{!}{=} \not{p} + M$$

- FOR MASSLESS SPINOR $(M=0)$ $(M \ll E)$

$$\sum_s u(p,s)\,\bar{u}(p,s) = \not{p}$$

ANALOGOUSLY FOR ANTI-PARTICLE SPINOR $(M=0)$

$$\sum_s v(p\,s)\,\bar{v}(p,s) = \not{p}$$

- TRACE THEOREM

$$\mathrm{Tr}\left\{ \not{a} \not{b} \not{c} \not{d} \right\} = 4 \left\{ a.b\ c.d - a.c\ b.d + a.d\ b.c \right.$$

PROOF

↳ $\gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu = 2 g^{\mu\nu}$

$$\downarrow \quad \times\ a_\mu\ b_\nu$$

$$\not{a}\not{b} + \not{b}\not{a} = 2\, a.b$$

↳ $\mathrm{Tr}(\not{a}\not{b}) = a.b\ \mathrm{Tr}\left(\mathbb{I}_{4\times 4} \right) = 4\, a.b$

↳ $\mathrm{Tr}\left\{ \not{a}\not{b}\not{c}\not{d} \right\}$

$$= 2\, a.b\ \underbrace{\mathrm{Tr}\left\{ \not{c}\not{d} \right\}}_{4\, c.d} - \mathrm{Tr}\left\{ \not{b}\not{a}\not{c}\not{d} \right\}$$

$$= 8 (a.b)(c.d) - 2\, a.c\ \mathrm{Tr}\left\{ \not{b}\not{d} \right\} + \mathrm{Tr}\left\{ \not{b}\not{c}\not{a}\not{d} \right\}$$

$$= 8 (a.b)(c.d) - 8 (a.c)(b.d) + 2\, a.d\ \mathrm{Tr}\left\{ \not{b}\not{c} \right\}$$

$$- \mathrm{Tr}\left\{ \not{b}\not{c}\not{d}\not{a} \right\}.$$

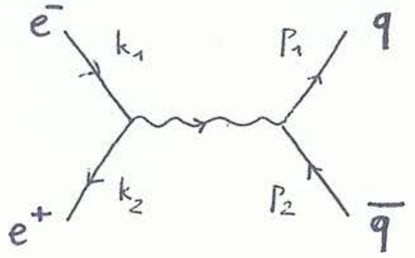
$$\downarrow$$

$$\mathrm{Tr}\left\{ \not{a}\not{b}\not{c}\not{d} \right\} + \mathrm{Tr}\left\{ \not{b}\not{c}\not{d}\not{a} \right\}$$

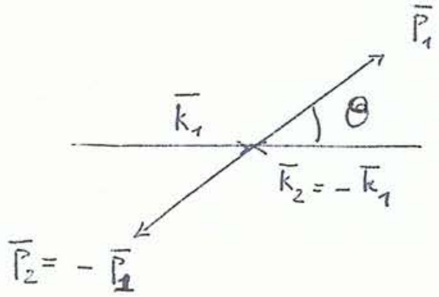
$$= 2 \left\{ 4 (a.b)(c.d) - 4 (a.c)(b.d) + 4 (a.d)(b.c) \right\}$$

$$= 2\ \mathrm{Tr}\left\{ \not{a}\not{b}\not{c}\not{d} \right\}.$$

⇒ CROSS SECTION FOR $e^+e^- \to q\bar{q}$



IN C.M.



$$|\bar{P}_1| = |\bar{k}_1| = \frac{\sqrt{s}}{2}$$

$$k_1\left(\frac{\sqrt{s}}{2}, \bar{k}_1\right) \qquad P_1\left(\frac{\sqrt{s}}{2}, \bar{P}_1\right)$$

$$k_2\left(\frac{\sqrt{s}}{2}, -\bar{k}_1\right) \qquad P_2\left(\frac{\sqrt{s}}{2}, -\bar{P}_1\right)$$

$$d\sigma(e^+e^- \to q\bar{q}) = \frac{1}{(2k_1^0)(2k_2^0)\underbrace{v_{rel}}_{2}} \cdot \int \frac{d^3\bar{P}_1}{(2\pi)^3 2P_1^0} \int \frac{d^3\bar{P}_2}{(2\pi)^3 2P_2^0} (2\pi)^4 \delta^4(k_1 + k_2 - P_1 - P_2)$$

$$\cdot\, e^4 e_q^2 \; \frac{1}{4} \sum_{s_{e^-} s_{e^+}} \sum_{s_q s_{\bar{q}}} \frac{1}{s^2} \Big| \bar{v}(k_2\, s_{e^+})\, \gamma_\mu\, u(k_1, s_{e^-}) \cdot \bar{u}(P_1, s_q)\, \gamma^\mu\, v(P_2, s_{\bar{q}}) \Big|^2$$

$$d\sigma = \frac{1}{2s} \cdot \frac{1}{(2\pi)^2} \int d\Omega_{\bar{P}_1} \frac{d|\bar{P}_1|\,|\bar{P}_1|^2}{4(P_1^0)^2} \underbrace{\delta(\sqrt{s} - 2|\bar{P}_1|)}_{\frac{1}{2}\delta(|\bar{P}_1| - \sqrt{s}/2)}$$

$$\cdot\, e^4 \cdot e_q^2 \; \frac{1}{4} \frac{1}{s^2} \cdot \underbrace{\mathrm{Tr}\left\{ \not{k}_2 \gamma_\mu \not{k}_1 \gamma_\nu \right\}}_{4\{k_{1\mu} k_{2\nu} + k_{1\nu} k_{2\mu} - (k_1 \cdot k_2) g_{\mu\nu}\}} \cdot \mathrm{Tr}\left\{ \not{P}_1 \gamma^\mu \not{P}_2 \gamma^\nu \right\}$$

$$\mathrm{Tr}\left\{ \not{P}_1 \gamma^\mu \not{P}_2 \gamma^\nu \right\} \to 4\left\{ P_1^\mu P_2^\nu + P_1^\nu P_2^\mu - (P_1 \cdot P_2) g^{\mu\nu} \right\}$$

$$\frac{d\sigma}{d\Omega_{P_1}} = \frac{1}{2s}\cdot\frac{1}{(2\pi)^2}\cdot\frac{1}{4}\cdot\frac{1}{2}$$

$$\cdot\, e^4 e_q^2 \frac{4}{s^2}\left\{2(k_1\cdot P_1)(k_2\cdot P_2) + 2(k_1\cdot P_2)(k_2\cdot P_1) - 2(k_1\cdot k_2)(P_1\cdot P_2) + 2(k_1\cdot k_2)(P_1\cdot P_2)\right\}$$

↓

$$k_1\cdot P_1 = k_2\cdot P_2 = \left(\frac{\sqrt{s}}{2}\right)^2 (1-\cos\Theta)$$

$$k_1\cdot P_2 = k_2\cdot P_1 = \left(\frac{\sqrt{s}}{2}\right)^2 (1+\cos\Theta)$$

$$\frac{d\sigma}{d\Omega_{P_1}} = \frac{1}{(2\pi)^2}\cdot e^4 e_q^2 \frac{1}{16s}\frac{2}{4}\left\{(1-\cos\Theta)^2 + (1+\cos\Theta)^2\right\}$$

↓ AXIAL SYMMETRY

$$\frac{d\sigma}{d\cos\Theta} = \frac{1}{2\pi}\cdot e^4 e_q^2\cdot\frac{1}{16s}(1+\cos^2\Theta)$$

$$\frac{d\sigma}{d\cos\Theta}(e^+e^-\to q\bar{q}) = \frac{(4\pi\alpha)^2}{2\pi}\frac{1}{16s}(1+\cos^2\Theta)\sum_q e_q^2\cdot N_c$$

↑ # COLORS OF PRODUCED q

DIFFERENTIAL CROSS SECTION

$$\frac{d\sigma}{d\cos\theta}(e^+e^- \to q\bar{q}) = \underbrace{\frac{\pi\alpha^2}{2s}(1+\cos^2\theta)}_{\frac{d\sigma}{d\cos\theta}(e^+e^- \to \mu^+\mu^-)} \quad N_c \sum_q e_q^2$$

$(1+\cos^2\theta)$ FACTOR IS CONSEQUENCE OF SPIN 1/2 NATURE OF QUARKS !

$$\frac{d\sigma}{d\cos\theta}(e^+e^- \to 2\ \text{JETS}) \sim (1+\cos^2\theta)$$

SAME ANGULAR DISTR AS UNDERLYING $e^+e^- \to q\bar{q}$ CROSS SECTION

TOTAL CROSS SECTION

$$\sigma_{total} = \int d\cos\theta \, \frac{d\sigma}{d\cos\theta}$$

$$\downarrow \qquad \int_{-1}^{1} d\cos\theta\,(1+\cos^2\theta) = \frac{8}{3}$$

$$\sigma_{tot}(e^+e^- \to q\bar{q}) = \underbrace{\frac{4\pi\alpha^2}{3s}}_{\sigma_{tot}(e^+e^- \to \mu^+\mu^-)} \cdot N_c \sum_q e_q^2$$

$$R \equiv \frac{\sigma_{tot}(e^+e^- \to q\bar{q})}{\sigma_{tot}(e^+e^- \to \mu^+\mu^-)} = N_c \cdot \sum_q e_q^2$$

$\Rightarrow$ SPIN-0 QUARK RESULT

$$\mathcal{M}_{fi} \sim \frac{1}{s} \; \bar{v}(k_2, s_{e+}) \, \gamma_\mu \, u(k_1, s_{e-}) \cdot (P_1 - P_2)^\mu$$

$$d\sigma : \frac{1}{4} \mathrm{Tr} \left\{ \not{P}_1 \gamma^\mu \not{P}_2 \gamma^\nu \right\}$$

REPLACED BY

$$(P_1 - P_2)^\mu \; (P_1 - P_2)^\nu$$

$$\Rightarrow \left\{ k_{1\mu} k_{2\nu} + k_{1\nu} k_{2\mu} - (k_1 \cdot k_2) \, g_{\mu\nu} \right\} \cdot (P_1 - P_2)^\mu (P_1 - P_2)^\nu$$

$$\downarrow \qquad P_1 - P_2 = 2P_1 - (P_1 + P_2)$$

$$\{ \quad \} \, (P_1 + P_2)^\mu = 0$$

$$= 4 \left\{ 2 (P_1 \cdot k_1)(P_1 \cdot k_2) - (k_1 \cdot k_2) \not{P_1^2} \right\} \quad (P_1^2 \to 0)$$

$$= 8 \left(\frac{s}{4} \right)^2 (1 - \cos\theta)(1 + \cos\theta)$$

$$= \frac{s^2}{2} (1 - \cos^2\theta) = \frac{s^2}{2} \sin^2\theta$$

$e^+e^- \rightarrow$ 2 JETS

$e^+e^- \rightarrow q\bar{q} \rightarrow$ 2 JETS

TASSO ⇒ PETRA e^+e^- R
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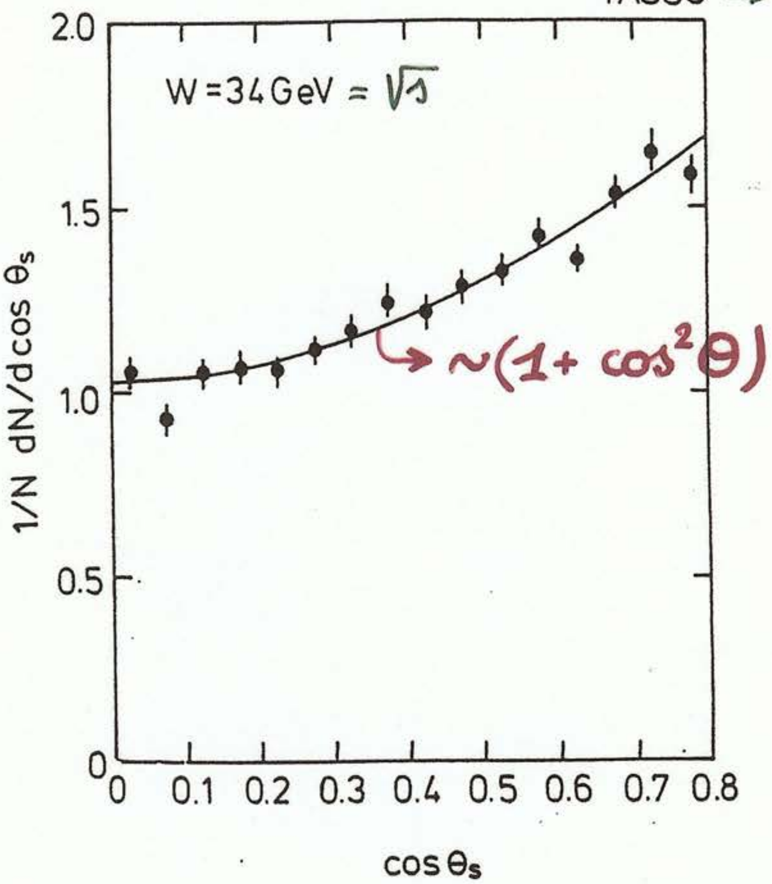


SPIN $\frac{1}{2}$ QUARKS : $\frac{d\sigma}{d\cos\theta} \sim (1 + \cos^2\theta)$

SPIN 0 QUARKS : $\frac{d\sigma}{d\cos\theta} \sim \sin^2\theta$

⇒ RATIONALE FOR COLOR QUANTUM NUMBER

$$R = \frac{\sigma_{tot}(e^+e^- \to q\bar{q})}{\sigma_{tot}(e^+e^- \to \mu^+\mu^-)} = \sum_q e_q^2$$

↑ IF EACH QUARK (u, d, s, c, b, t) COMES IN ONE "CHARGE" STATE

* e.g. FOR $\sqrt{s} < M_{J/\psi}$

ONLY $e^+e^- \to u\bar{u}$

$\to d\bar{d}$

$\to s\bar{s}$ POSSIBLE

$$R = e_u^2 + e_d^2 + e_s^2 = \frac{4}{9} + \frac{1}{9} + \frac{1}{9} = \frac{2}{3}$$

EXPERIMENTALLY ONE OBSERVES $R = 2$!

* FOR $\sqrt{s} > M_{\Upsilon}$ BUT $\sqrt{s} < t\bar{t}$ THRESHOLD

$e^+e^- \to u\bar{u}, d\bar{d}, s\bar{s}, c\bar{c}, b\bar{b}$ POSSIBLE

$$R = e_u^2 + e_d^2 + e_s^2 + e_c^2 + e_b^2$$

$$= \frac{4}{9} + \frac{1}{9} + \frac{1}{9} + \frac{4}{9} + \frac{1}{9} = \frac{11}{9}$$

EXP. ONE OBERVES $R = \frac{11}{3}$

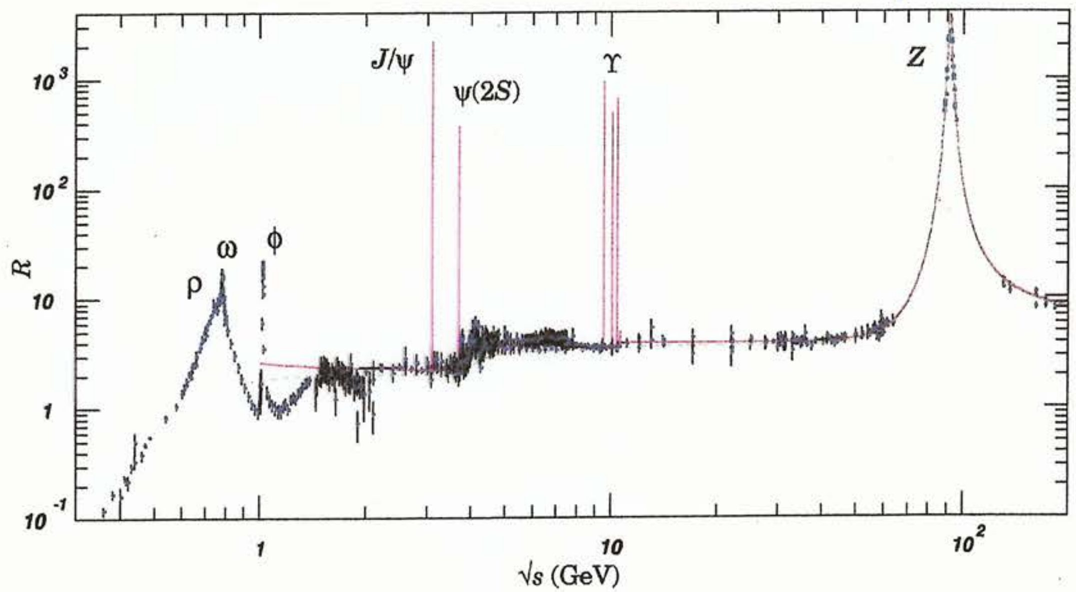
∴ EACH QUARK COMES IN 3 COLORS $(N_c = 3)$

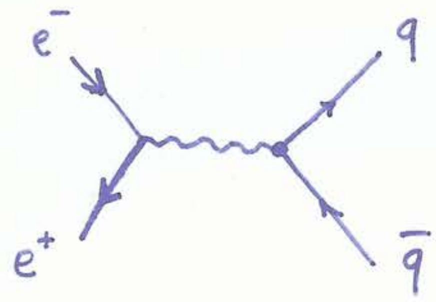
$$R = N_c \cdot \sum_q e_q^2$$

- Number of colours (N_c) can be tested in experiment:

e.g. $$R = \frac{\sigma[e^+e^- \to \text{hadrons}]}{\sigma[e^+e^- \to \mu^+\mu^-]} \propto N_c \quad \text{[see below]} \qquad (11)$$

$$= N_c \sum_q e_q^2$$





also $d\sigma[pp \to \mu^+\mu^- X]$, $\Gamma[\pi^0 \to \gamma\gamma]$

$\Rightarrow$ all measurements imply $N_c = 3.0 \pm \ldots$ $\checkmark$