Exercise sheet 10
Theoretical Physics 5 : WS 2019/2020
Lecturers : Prof. M. Vanderhaeghen, Dr. I. Danilkin

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Exercise 0.

How much time did it take to complete the task?

Exercise 1. (30 points)

Consider the following integral:

$$I = \int \frac{\mathrm{d}^3\vec{p}}{(2\pi)^3 2E_p} \frac{\mathrm{d}^3\vec{q}}{(2\pi)^3 2E_q} (2\pi)^4 \delta^{(3)}(\vec{p}+\vec{q}-\vec{P})\delta(E_p + E_q - P^0),$$

where $E_p^2 = m^2 + \vec{p}^{\,2}$ and $E_q^2 = m'^2 + \vec{q}^{\,2}$.
Show that the integral I is Lorentz invariant. Calculate it in the frame where $\vec{P} = 0$.

Exercise 2. (30 points)

Prove that the differential cross section for a $2 \longrightarrow 2$ scattering in the centre-of-mass frame can be written as:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{1}{64\pi^2 s} \frac{|\vec{p}_f|}{|\vec{p}_i|} |\mathcal{M}|^2,$$

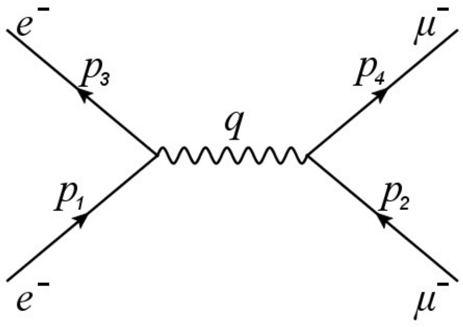
where $\vec{p}_i$ and $\vec{p}_f$ are the centre-of-mass momenta before and after the scattering, and s is the corresponding total energy of the system.

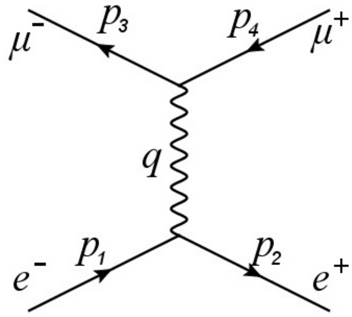
Exercise 3. (20 points)

A beam of alpha particles of energy $T = 0.1$ GeV collides against a fixed target of aluminium (density $\rho = 2.7$ g/cm^3, molar mass $A = 27$ g/mol) of thickness of $d = 1$ cm. The beam flux at the target is $\Phi = 10^9$ s^{-1}. A scintillating detector is placed at an angle $\theta = 30°$ from the beam axis, and $L = 1$ m away from the target. The active surface of the detector has a cross section of 1 cm $\times$ 1 cm as seen from the target. Estimate the counting rate (a rate of counts per unit time) measured by the detector.

Exercise 4. (20 points)

Consider $e^-\mu^-$ scattering and e^+e^- annihilation into $\mu^+\mu^-$ processes (the arrows denote the direction of negative charge flow):





Show which of them corresponds to a space-like momentum transfer ($q^2 = q^\mu q_\mu < 0$) and which to a time-like one ($q^2 > 0$).