Aus: J.D. Jackson, Classical Electrodynamics, Wiley & Sons, New York [u.a.], 1975

3 Various Systems of Electromagnetic Units

The various systems of electromagnetic units differ in their choices of the magnitudes and dimensions of the various constants above. Because of relations (A.5) and (A.11) there are only two constants (e.g., k_1, k_3) that can (and must) be chosen arbitrarily. It is convenient, however, to tabulate all four constants (k_1, k_2, α, k_3) for the commoner systems of units. These are given in Table 1. We note that, apart from dimensions, the em units and MKSA units are very similar, differing only in various powers of 10 in their mechanical and electromagnetic units. The Gaussian and Heaviside-Lorentz systems differ only by factors of 4π. Only in the Gaussian (and Heaviside-Lorentz) system does k_3 have dimensions. It is evident from (A.7) that, with k_3 having dimensions of a reciprocal velocity, $\mathbf{E}$ and $\mathbf{B}$ have the same dimensions. Furthermore, with $k_3 = c^{-1}$, (A.7) shows that for electromagnetic waves in free space $\mathbf{E}$ and $\mathbf{B}$ are equal in magnitude as well.

Only electromagnetic fields in free space have been discussed so far. Consequently only the two fundamental fields $\mathbf{E}$ and $\mathbf{B}$ have appeared. There remains the task of defining the macroscopic field variables $\mathbf{D}$ and $\mathbf{H}$. If the averaged electromagnetic properties of a material medium are described by a macroscopic polarization $\mathbf{P}$ and a magnetization $\mathbf{M}$, the general form of the definitions of $\mathbf{D}$ and $\mathbf{H}$ are

$$\left.\begin{aligned} \mathbf{D} &= \epsilon_0 \mathbf{E} + \lambda \mathbf{P} \\ \mathbf{H} &= \frac{1}{\mu_0} \mathbf{B} - \lambda' \mathbf{M} \end{aligned}\right\} \qquad (A.12)$$

where ϵ_0, μ_0, λ, λ' are proportionality constants. Nothing is gained by making $\mathbf{D}$ and $\mathbf{P}$ or $\mathbf{H}$ and $\mathbf{M}$ have different dimensions. Consequently λ and λ' are chosen as pure numbers ($\lambda = \lambda' = 1$ in rationalized systems, $\lambda = \lambda' = 4\pi$ in unrationalized systems). But there is the choice as to whether $\mathbf{D}$ and $\mathbf{P}$ will differ in dimensions from $\mathbf{E}$, and $\mathbf{H}$ and $\mathbf{M}$ differ from $\mathbf{B}$. This choice is made for convenience and simplicity, usually in order to make the macroscopic Maxwell equations have a relatively simple, neat form. Before tabulating the choices made for different systems, we note that for linear, isotropic media the constitutive relations are always written

$$\left.\begin{aligned} \mathbf{D} &= \epsilon \mathbf{E} \\ \mathbf{B} &= \mu \mathbf{H} \end{aligned}\right\} \qquad (A.13)$$

Thus in (A.12) the constants ϵ_0 and μ_0 are the vacuum values of ϵ and μ. The relative permittivity of a substance (often called the *dielectric constant*) is defined as the dimensionless ratio (ϵ/ϵ_0), while the relative permeability (often called the *permeability*) is defined as (μ/μ_0).

Table 2 displays the values of ϵ_0 and μ_0, the defining equations for $\mathbf{D}$ and $\mathbf{H}$, the macroscopic forms of the Maxwell equations, and the Lorentz force equation in the five common systems of units of Table 1. For each system of units the continuity equation for charge and current is given by (A.1), as can be verified from the first pair of the Maxwell equations in the table in each case.* Similarly, in all systems the statement of Ohm's law is $\mathbf{J} = \sigma \mathbf{E}$, where σ is the conductivity.

Table 1
Magnitudes and Dimensions of the Electromagnetic Constants for Various Systems of Units

The dimensions are given after the numerical values. The symbol c stands for the velocity of light in vacuum ($c = 2.998 \times 10^{10}$ cm/sec $= 2.998 \times 10^8$ m/sec). The first four systems of units use the centimeter, gram, and second as their fundamental units of length, mass, and time (l, m, t). The MKSA system uses the meter, kilogram, and second, plus current (I) as a fourth dimension, with the ampere as unit.

System	k_1	k_2	α	k_3
Electrostatic (esu)	1	$c^{-2}(t^2l^{-2})$	1	1
Electromagnetic (emu)	$c^2(l^2t^{-2})$	1	1	1
Gaussian	1	$c^{-2}(t^2l^{-2})$	$c(lt^{-1})$	$c^{-1}(tl^{-1})$
Heaviside-Lorentz	$\frac{1}{4\pi}$	$\frac{1}{4\pi c^2}(t^2l^{-2})$	$c(lt^{-1})$	$c^{-1}(tl^{-1})$
Rationalized MKSA	$\frac{1}{4\pi\epsilon_0} = 10^{-7}c^2$ $(ml^3t^{-4}I^{-2})$	$\frac{\mu_0}{4\pi} = 10^{-7}$ $(mlt^{-2}I^{-2})$	1	1

4 Conversion of Equations and Amounts between Gaussian Units and MKSA Units

The two systems of electromagnetic units in most common use today are the Gaussian and rationalized MKSA systems. The MKSA system has the virtue of overall convenience in practical, large-scale phenomena, especially in engineering applications. The Gaussian system is more suitable for microscopic problems involving the electrodynamics of individual charged particles, etc. Since microscopic, relativistic problems are important in this book, it has been found most convenient to use Gaussian units throughout. In Chapter 8 on wave guides and cavities an attempt has been made to placate the engineer by writing each key

* Some workers employ a modified Gaussian system of units in which current is defined by $I = (1/c)(dq/dt)$. Then the current density $\mathbf{J}$ in the table must be replaced by $c\mathbf{J}$, and the continuity equation is $\nabla \cdot \mathbf{J} + (1/c)(\partial\rho/\partial t) = 0$. See also the footnote below Table 4.

Table 2
Definitions of ϵ_0, μ_0, D, H, Macroscopic Maxwell Equations, and Lorentz Force Equation in Various Systems of Units

Where necessary the dimensions of quantities are given in parentheses. The symbol c stands for the velocity of light in vacuum with dimensions (lt^{-1}).

System	ϵ_0	μ_0	D, H	Macroscopic Maxwell Equations				Lorentz Force per Unit charge
Electrostatic (esu)	1	c^{-2} (t^2l^{-2})	$\mathbf{D}=\mathbf{E}+4\pi\mathbf{P}$ $\mathbf{H}=c^2\mathbf{B}-4\pi\mathbf{M}$	$\nabla\cdot\mathbf{D}=4\pi\rho$	$\nabla\times\mathbf{H}=4\pi\mathbf{J}+\frac{\partial\mathbf{D}}{\partial t}$	$\nabla\times\mathbf{E}+\frac{\partial\mathbf{B}}{\partial t}=0$	$\nabla\cdot\mathbf{B}=0$	$\mathbf{E}+\mathbf{v}\times\mathbf{B}$
Electromagnetic (emu)	c^{-2} (t^2l^{-2})	1	$\mathbf{D}=\frac{1}{c^2}\mathbf{E}+4\pi\mathbf{P}$ $\mathbf{H}=\mathbf{B}-4\pi\mathbf{M}$	$\nabla\cdot\mathbf{D}=4\pi\rho$	$\nabla\times\mathbf{H}=4\pi\mathbf{J}+\frac{\partial\mathbf{D}}{\partial t}$	$\nabla\times\mathbf{E}+\frac{\partial\mathbf{B}}{\partial t}=0$	$\nabla\cdot\mathbf{B}=0$	$\mathbf{E}+\mathbf{v}\times\mathbf{B}$
Gaussian	1	1	$\mathbf{D}=\mathbf{E}+4\pi\mathbf{P}$ $\mathbf{H}=\mathbf{B}-4\pi\mathbf{M}$	$\nabla\cdot\mathbf{D}=4\pi\rho$	$\nabla\times\mathbf{H}=\frac{4\pi}{c}\mathbf{J}+\frac{1}{c}\frac{\partial\mathbf{D}}{\partial t}$	$\nabla\times\mathbf{E}+\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t}=0$	$\nabla\cdot\mathbf{B}=0$	$\mathbf{E}+\frac{\mathbf{v}}{c}\times\mathbf{B}$
Heaviside-Lorentz	1	1	$\mathbf{D}=\mathbf{E}+\mathbf{P}$ $\mathbf{H}=\mathbf{B}-\mathbf{M}$	$\nabla\cdot\mathbf{D}=\rho$	$\nabla\times\mathbf{H}=\frac{1}{c}\left(\mathbf{J}+\frac{\partial\mathbf{D}}{\partial t}\right)$	$\nabla\times\mathbf{E}+\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t}=0$	$\nabla\cdot\mathbf{B}=0$	$\mathbf{E}+\frac{\mathbf{v}}{c}\times\mathbf{B}$
Rationalized MKSA	$\frac{10^7}{4\pi c^2}$ $(I^2t^4m^{-1}l^{-3})$	$4\pi\times10^{-7}$ $(mlI^{-2}t^{-2})$	$\mathbf{D}=\epsilon_0\mathbf{E}+\mathbf{P}$ $\mathbf{H}=\frac{1}{\mu_0}\mathbf{B}-\mathbf{M}$	$\nabla\cdot\mathbf{D}=\rho$	$\nabla\times\mathbf{H}=\mathbf{J}+\frac{\partial\mathbf{D}}{\partial t}$	$\nabla\times\mathbf{E}+\frac{\partial\mathbf{B}}{\partial t}=0$	$\nabla\cdot\mathbf{B}=0$	$\mathbf{E}+\mathbf{v}\times\mathbf{B}$

Table 3
Conversion Table for Symbols and Formulas

The symbols for mass, length, time, force, and other not specifically electromagnetic quantities are unchanged. To convert any equation in Gaussian variables to the corresponding equation in MKSA quantities, on both sides of the equation replace the relevant symbols listed below under "Gaussian" by the corresponding "MKSA" symbols listed on the right. The reverse transformation is also allowed. Since the length and time symbols are unchanged, quantities which differ dimensionally from one another only by powers of length and/or time are grouped together where possible.

Quantity	Gaussian	MKSA
Velocity of light	c	$(\mu_0\epsilon_0)^{-1/2}$
Electric field (potential, voltage)	$\mathbf{E}(\Phi, V)$	$\sqrt{4\pi\epsilon_0}\,\mathbf{E}(\Phi, V)$
Displacement	$\mathbf{D}$	$\sqrt{\frac{4\pi}{\epsilon_0}}\,\mathbf{D}$
Charge density (charge, current density, current, polarization)	$\rho(q, \mathbf{J}, I, \mathbf{P})$	$\frac{1}{\sqrt{4\pi\epsilon_0}}\rho(q, \mathbf{J}, I, \mathbf{P})$
Magnetic induction	$\mathbf{B}$	$\sqrt{\frac{4\pi}{\mu_0}}\,\mathbf{B}$
Magnetic field	$\mathbf{H}$	$\sqrt{4\pi\mu_0}\,\mathbf{H}$
Magnetization	$\mathbf{M}$	$\sqrt{\frac{\mu_0}{4\pi}}\,\mathbf{M}$
Conductivity	σ	$\frac{\sigma}{4\pi\epsilon_0}$
Dielectric constant	ϵ	$\frac{\epsilon}{\epsilon_0}$
Permeability	μ	$\frac{\mu}{\mu_0}$
Resistance (impedance)	$R(Z)$	$4\pi\epsilon_0 R(Z)$
Inductance	L	$4\pi\epsilon_0 L$
Capacitance	C	$\frac{1}{4\pi\epsilon_0}C$

Table 4
Conversion Table for Given Amounts of a Physical Quantity

The table is arranged so that a given amount of some physical quantity, expressed as so many MKSA or Gaussian units of that quantity, can be expressed as an equivalent number of units in the other system. Thus the entries in each row stand for the same amount, expressed in different units. All factors of 3 (apart from exponents) should, for accurate work, be replaced by (2.99792456), arising from the numerical value of the velocity of light. For example, in the row for displacement (D), the entry ($12\pi\times10^5$) is actually ($2.99792\times4\pi\times10^5$). Where a name for a unit has been agreed on or is in common usage, that name is given. Otherwise, one merely reads so many Gaussian units, or MKSA or SI units.

Physical Quantity	Symbol	Rationalized MKSA	Gaussian	
Length	l	1 meter (m)	10^2	centimeters (cm)
Mass	m	1 kilogram (kg)	10^3	grams (gm)
Time	t	1 second (sec)	1	second (sec)
Frequency	ν	1 hertz (Hz)	1	hertz (Hz)
Force	F	1 newton	10^5	dynes
Work, Energy	W, U	1 joule	10^7	ergs
Power	P	1 watt	10^7	ergs sec^{-1}
Charge	q	1 coulomb	3×10^9	statcoulombs
Charge density	ρ	1 coul m^{-3}	3×10^3	statcoul cm^{-3}
Current	I	1 ampere (amp)	3×10^9	statamperes
Current density	J	1 amp m^{-2}	3×10^5	statamp cm^{-2}
Electric field	E	1 volt m^{-1}	$\frac{1}{3}\times10^{-4}$	statvolt cm^{-1}
Potential	Φ, V	1 volt	$\frac{1}{300}$	statvolt
Polarization	P	1 coul m^{-2}	3×10^5	dipole moment cm^{-3}
Displacement	D	1 coul m^{-2}	$12\pi\times10^5$	statvolt cm^{-1} (statcoul cm^{-2})
Conductivity	σ	1 mho m^{-1}	9×10^9	sec^{-1}
Resistance	R	1 ohm	$\frac{1}{9}\times10^{-11}$	sec cm^{-1}
Capacitance	C	1 farad	9×10^{11}	cm
Magnetic flux	ϕ, F	1 weber	10^8	gauss cm^2 or maxwells
Magnetic induction	B	1 tesla	10^4	gauss
Magnetic field	H	1 ampere-turn m^{-1}	$4\pi\times10^{-3}$	oersted
Magnetization	M	1 ampere m^{-1}	10^{-3}	magnetic moment cm^{-3}
*Inductance	L	1 henry	$\frac{1}{9}\times10^{-11}$	

* There is some confusion prevalent about the unit of inductance in Gaussian units. This stems from the use by some authors of a modified system of Gaussian units in which current is measured in electromagnetic units, so that the connection between charge and current is $I_m=(1/c)(dq/dt)$. Since inductance is defined through the induced voltage $V=L(dI/dt)$ or the energy $U=\frac{1}{2}LI^2$, the choice of current defined in Section 2 means that our Gaussian unit of inductance is equal in magnitude and dimensions (t^2l^{-1}) to the electrostatic unit of inductance. The electromagnetic current I_m is related to our Gaussian current I by the relation $I_m=(1/c)I$. From the energy definition of inductance we see that the electromagnetic inductance L_m is related to our Gaussian inductance L through $L_m=c^2L$. Thus L_m has the dimensions of length. The modified Gaussian system generally uses the electromagnetic unit of inductance, as well as current. Then the voltage relation reads $V=(L_m/c)(dI_m/dt)$. The numerical connection between units of inductance is

$$1\text{ henry}=\tfrac{1}{9}\times10^{-11}\text{ Gaussian (es) unit}=10^9\text{ emu}$$

formula in such a way that omission of the factor in square brackets in the equation will yield the equivalent MKSA equation (provided all symbols are reinterpreted as MKSA variables).

Tables 3 and 4 are designed for general use in conversion from one system to the other. Table 3 is a conversion scheme for *symbols and equations* which allows the reader to convert any equation from the Gaussian system to the MKSA system and vice versa. Simpler schemes are available for conversion only *from* the MKSA system *to* the Gaussian system, and other general schemes are possible. But by keeping all mechanical quantities unchanged, the recipe in Table 3 allows the straightforward conversion of quantities which arise from an interplay of electromagnetic and mechanical forces (e.g., the fine structure constant $e^2/\hbar c$ and the plasma frequency $\omega_p^2=4\pi ne^2/m$) without additional considerations. Table 4 is a conversion table for units to allow the reader to express a given amount of any physical entity as a certain number of MKSA units or cgs-Gaussian units.