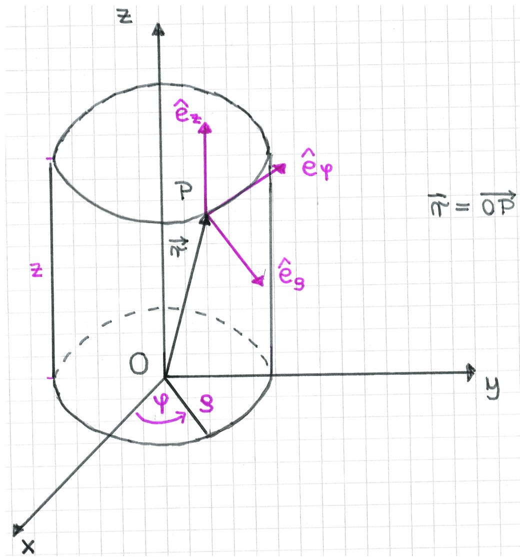
S. Scherer 15. November 2019

Mathematische Rechenmethoden 1 (B.Ed.) WiSe 2019/2020
Zylinder- und Kugelkoordinaten

1. Zylinderkoordinaten



$$
\begin{aligned}
x &= \rho\cos(\varphi),\\
y &= \rho\sin(\varphi),\\
z &= z,\\
0 \le \rho &= \sqrt{x^2+y^2} < \infty,\\
\varphi &\in [0, 2\pi[,\\
-\infty &< z < \infty,\\
\hat{e}_\rho &= \cos(\varphi)\hat{e}_x + \sin(\varphi)\hat{e}_y,\\
\hat{e}_\varphi &= -\sin(\varphi)\hat{e}_x + \cos(\varphi)\hat{e}_y,\\
\hat{e}_z &= \hat{e}_z.
\end{aligned}
$$

$(\hat{e}_\rho, \hat{e}_\varphi, \hat{e}_z)$ bildet ein rechtshändiges System.

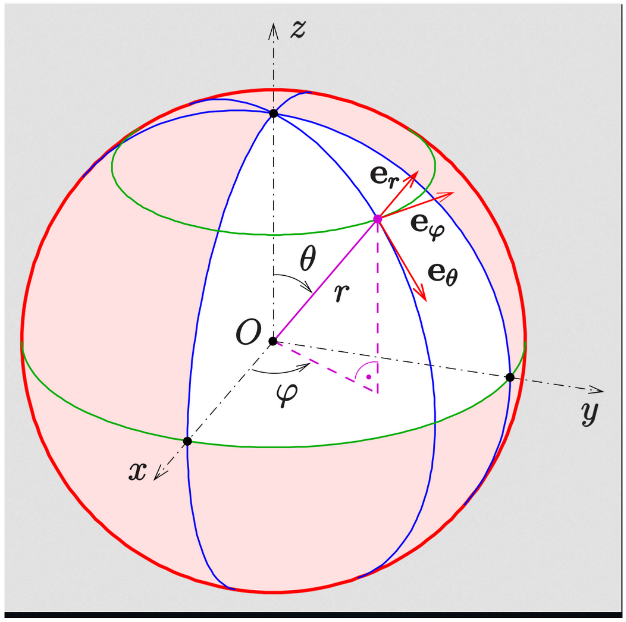
Umkehrung:

$$
\begin{aligned}
\hat{e}_x &= \cos(\varphi)\hat{e}_\rho - \sin(\varphi)\hat{e}_\varphi,\\
\hat{e}_y &= \sin(\varphi)\hat{e}_\rho + \cos(\varphi)\hat{e}_\varphi.
\end{aligned}
$$

Probe:

$$
\begin{aligned}
\vec{r} &= \rho\hat{e}_\rho + z\hat{e}_z\\
&= \rho\left[\cos(\varphi)\hat{e}_x + \sin(\varphi)\hat{e}_y\right] + z\hat{e}_z\\
&= x\hat{e}_x + y\hat{e}_y + z\hat{e}_z.
\end{aligned}
$$

2. Kugelkoordinaten



Quelle: Wikimedia Commons,
Autor: Ag2gaeh, CC BY-SA 4.0

$$
\begin{aligned}
x &= r\sin(\theta)\cos(\varphi),\\
y &= r\sin(\theta)\sin(\varphi),\\
z &= r\cos(\theta),\\
0 \le r &= \sqrt{x^2+y^2+z^2} < \infty,\\
\varphi &\in [0, 2\pi[,\\
\theta &\in [0,\pi], \quad \cos(\theta) = \frac{z}{r},\\
\hat{e}_r &= \sin(\theta)\cos(\varphi)\hat{e}_x + \sin(\theta)\sin(\varphi)\hat{e}_y + \cos(\theta)\hat{e}_z,\\
\hat{e}_\theta &= \cos(\theta)\cos(\varphi)\hat{e}_x + \cos(\theta)\sin(\varphi)\hat{e}_y - \sin(\theta)\hat{e}_z,\\
\hat{e}_\varphi &= -\sin(\varphi)\hat{e}_x + \cos(\varphi)\hat{e}_y.
\end{aligned}
$$

$(\hat{e}_r, \hat{e}_\theta, \hat{e}_\varphi)$ bildet ein rechtshändiges System.

Umkehrung:

$$
\begin{aligned}
\hat{e}_x &= \sin(\theta)\cos(\varphi)\hat{e}_r + \cos(\theta)\cos(\varphi)\hat{e}_\theta - \sin(\varphi)\hat{e}_\varphi,\\
\hat{e}_y &= \sin(\theta)\sin(\varphi)\hat{e}_r + \cos(\theta)\sin(\varphi)\hat{e}_\theta + \cos(\varphi)\hat{e}_\varphi,\\
\hat{e}_z &= \cos(\theta)\hat{e}_r - \sin(\theta)\hat{e}_\theta.
\end{aligned}
$$

Probe:

$$
\begin{aligned}
\vec{r} &= r\hat{e}_r\\
&= r\left[\sin(\theta)\cos(\varphi)\hat{e}_x + \sin(\theta)\sin(\varphi)\hat{e}_y + \cos(\theta)\hat{e}_z\right]\\
&= x\hat{e}_x + y\hat{e}_y + z\hat{e}_z.
\end{aligned}
$$