Symmetrien in der Physik

PD Dr. Georg von Hippel

JOHANNES GUTENBERG UNIVERSITÄT MAINZ

Wintersemester 2019/2020

Organisatorisches

JOGUStINe – Anmeldung

Die Anmeldephase endet am **Freitag, 18.10.2019 um 21:00 Uhr**.

Alle Teilnehmer, die Credit-Points für diesen Kurs erhalten wollen, müssen sich bis dahin angemeldet haben.

Organisatorisches

Leistungsnachweis

Als Leistungsnachweis ist eine **mündliche Prüfung** von 30 Minuten vorgesehen.

Voraussetzung für die Prüfungszulassung ist die erfolgreiche Teilnahme an den Übungen.

Organisatorisches

Übungen

Übungen finden in der Regel jede zweite Woche Donnerstags 10:00–12:00 Uhr im Galilei-Raum statt.

Übungstermine: 24.10., 7.11., 21.11., 5.12., 19.12., 16.01., 30.01.

Übungsblätter werden in der Übung aus- und abgegeben.
(Das erste Übungsblatt gibt es in der nächsten Vorlesung.)

Organisatorisches

Vorlesungstermine

Montags beginnt die Vorlesung um **08:30** und geht ohne Pause durch.

Am **06.01.** und **09.01.** fällt die Vorlesung voraussichtlich aus.

Die ausgefallenen Termine werden am Semesterende nach Vereinbarung nachgeholt.

Website zur Vorlesung

https://wwwth.kph.uni-mainz.de/ws201920-symmetrien/

Literatur zur Vorlesung

S. Scherer, *Symmetrien und Gruppen in der Teilchenphysik*, Springer (Berlin/Heidelberg) 2016.

H.F. Jones, *Groups, Representations and Physics*, IoP Publishing (Bristol/Philadelphia) 1998.

A. Zee, *Group Theory in a Nutshell for Physicists*, Princeton University Press 2016.

W. Greiner, *Theoretische Physik (Bd. 5: Quantenmechanik II – Symmetrien)*, Harri Deutsch (Thun/Frankfurt a.M.) 1985.

What is Symmetry?

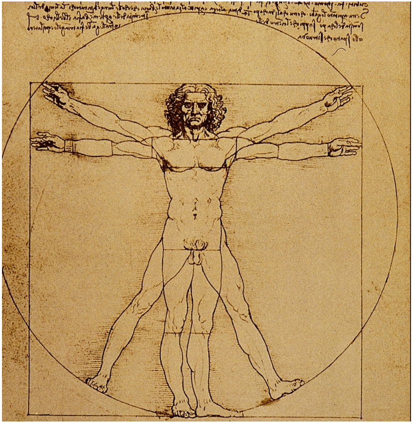
Etymology from gr. συν- “with-, together-” and μέτρον “measure” ⇝ συμμετρία “regularity, (proper) proportion”

The corresponding Latin roots *con-* “with-, together-” and *mensura* “measure” produce with *-abilis, -ibilis* “-able” modern *commensurable* “measurable by the same standard, proportionate”

Originally, the word “symmetry” thus means more or less regularity or proportionality.

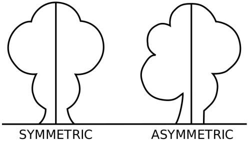
What is Symmetry?

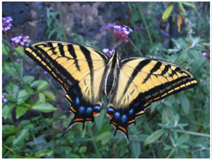
In English, the word "symmetry" first occurs as an architectural term of art referring to a harmony of parts or proportions (1600-1800, first attested 1563 [OED]).

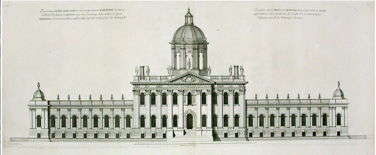


What is Symmetry?

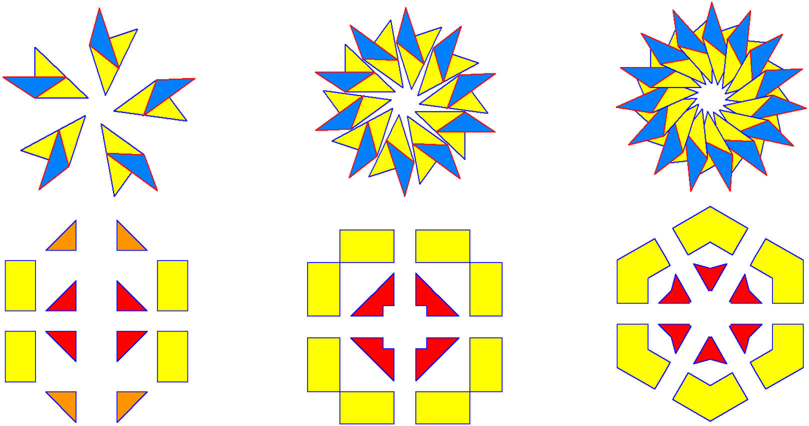
In modern **colloquial use** "symmetry" generally refers to the equable distribution of parts about a dividing line or centre.







Typical **high-school definition**: A figure is symmetric, if it can be decomposed into two or more mutually congruent parts, which are arranged in a systematic fashion.



What is Symmetry?

Hermann Weyl
(1885–1955)

Mathematical notion of symmetry

after Hermann Weyl:
Symmetry means invariance under a group of automorphisms.

Richard Feynman
(1918–1988)

Richard Feynman:
“Professor Weyl, the mathematician, gave an excellent definition of symmetry, which is that a thing is symmetrical if there is something that you can do to it so that after you have finished doing it it looks the same as it did before.”

What is Symmetry?

A bit more formally: The subset S of a space R is symmetric under $f \in \mathrm{Aut}(R)$ if $f(S) = S$.

The maps under which a given S is symmetric, form a (concrete) group:

$$\mathrm{id}_X(S) = S \tag{1}$$

$$f(S) = S \Rightarrow f^{-1}(S) = f^{-1}(f(S)) = S \tag{2}$$

$$f_1(S) = S \wedge f_2(S) = S \Rightarrow (f_1 \circ f_2)(S) = f_1(f_2(S)) = S \tag{3}$$

Symmetry in the mathematical sense is thus closely related to **group theory**.

What is Symmetry?

Similarly, an equation on R is symmetric under $f \in \mathrm{Aut}(R)$ if $f(x)$ satisfies it iff x does.

In classical mechanics, the invariance of the equations of motion can be expressed as the invariance of the action:

$$\int L[q(t)]\,\mathrm{d}t = \int L[f[q](t)]\,\mathrm{d}t.$$

In quantum mechanics, the invariance of the Schrödinger equation under (time-independent) unitary transformations $\hat{U} = \mathrm{e}^{i\alpha\hat{Q}}$ of the Hilbert space corresponds to the vanishing of the commutator with the Hamiltonian:

$$[\hat{H}, \hat{Q}] = 0.$$

Literature on the Notion of Symmetry

B. Krimmel (Hrsg.), *Symmetrie in Kunst, Natur und Wissenschaft*, Ausstellungskatalog, Institut Mathildenhöhe (Darmstadt) 1986.

R. Wille (Hrsg.), *Symmetrie in Geistes- und Naturwissenschaft*, Tagungsband, Springer (Berlin/Heidelberg) 1986.

H. Weyl, *Symmetry*, Princeton University Press 1952.
[dt.: *Symmetrie*, Birkhäuser (Basel) 1955]

R. Feynman, *The Character of Physical Law*, MIT Press 1965.
[dt.: *Vom Wesen physikalischer Gesetze*, Piper (München) 1990.]

Fundamental Symmetries in Physics

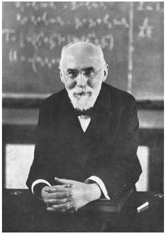
Symmetry is a necessary requirement in order to even be able to speak of laws of Nature – the laws have to be time-independent!

$\rightsquigarrow$ Time translation invariance: $t \mapsto t' = t + \Delta t$, $\Delta t \in \mathbb{R}$

Moreover we observe or postulate the homogeneity and isotropy of space.

$\rightsquigarrow$ Translation invariance: $\boldsymbol{x} \mapsto \boldsymbol{x}' = \boldsymbol{x} + \boldsymbol{a}$, $\boldsymbol{a} \in \mathbb{R}^3$
Rotational invariance: $\boldsymbol{x} \mapsto D\boldsymbol{x}$, $D \in \mathrm{SO}(3)$

Fundamental Symmetries in Physics



Hendrik Antoon Lorentz (1853–1928)

Henri Poincaré (1854–1912)

From the principle of relativity, we infer the invariance of physical laws under a change of inertial reference frame.

$\rightsquigarrow$ Boost invariance:
$$\boldsymbol{x} \mapsto \boldsymbol{x}' = \boldsymbol{x} - (\boldsymbol{x}\cdot\hat{\boldsymbol{v}})\hat{\boldsymbol{v}} + \gamma(\boldsymbol{v})((\boldsymbol{x}\cdot\hat{\boldsymbol{v}})\hat{\boldsymbol{v}} - t\boldsymbol{v}),$$
$$t \mapsto t' = \gamma(\boldsymbol{v})\left(t - \frac{1}{c^2}\boldsymbol{v}\cdot\boldsymbol{x}\right)$$

with $\gamma(\boldsymbol{v}) = 1/\sqrt{1-\boldsymbol{v}^2/c^2}$, $\boldsymbol{v}\in\mathbb{R}^3$

Taken together, rotations and boost form the **Lorentz group**, with translations the **Poincaré group**.

Fundamental Symmetries in Physics

Time reversal $T : t \mapsto -t$, parity $P : \boldsymbol{x} \mapsto -\boldsymbol{x}$ and charge conjugation $C : e \mapsto -e$ (swapping particles and antiparticles) are symmetries of classical mechanics and electromagnetism.

In particle physics, these symmetries are violated by the weak interaction.

However, the **CPT-Theorem** states that the combination of all three operations must be a symmetry of any local quantum field theory.

Among other things, this guarantees that particles and antiparticles have the same mass.

Discrete and Continuous Symmetries

C, P and T are discrete symmetries, whereas rotations, boosts and translations are parameterized by continuous parameters.

Sophus Lie
(1842–1899)

The rotation group, the Lorentz group and the Poincaré group are examples of **Lie groups**, i.e. groups which are also manifolds in a manner that is compatible with the group structure, i.e. such that the group multiplication and the inverse are smooth.

By considering infinitesimal transformations, Lie groups can be described in terms of their **Lie algebras**.

Symmetries and Conservation Laws

Emmy Noether
(1882–1935)

Noether-Theorem: Every symmetry corresponds to a conserved quantity.

For continuous symmetries $f(q) = q + \epsilon\,\delta q$ this follows from

$$L(q,\dot q) = L(q+\epsilon\,\delta q, \dot q + \epsilon\,\delta\dot q)$$
$$= L(q,\dot q) + \epsilon \underbrace{\left(\frac{\partial L}{\partial q}\delta q + \frac{\partial L}{\partial \dot q}\delta\dot q\right)}_{=0}$$
$$\Rightarrow 0 = \left(\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \dot q}\delta q + \frac{\partial L}{\partial \dot q}\delta\dot q\right)$$
$$= \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial \dot q}\delta q\right)$$

Symmetries and Conservation Laws

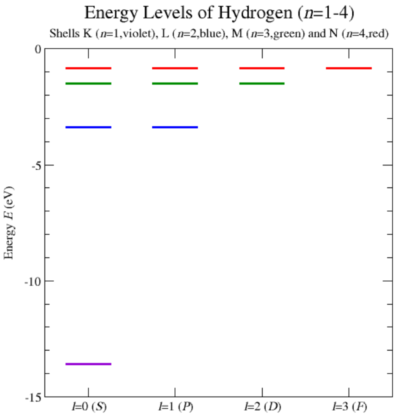
Emmy Noether
(1882–1935)

Noether-Theorem: Every symmetry corresponds to a conserved quantity.

For example invariance under
time translations implies energy conservation,
spatial translations implies momentum conservation,
spatial rotations implies angular momentum conservation.

Symmetries and Degenerate Energy Levels

In quantum mechanics, symmetry leads to a degeneracy of states, since $[\hat{H}, \hat{Q}] = 0$ and $\hat{H}\,|\psi\rangle = E\,|\psi\rangle$ imply $\hat{H}\left(\hat{Q}\,|\psi\rangle\right) = E\left(\hat{Q}\,|\psi\rangle\right)$.



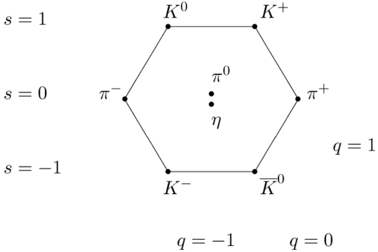
E.g. the "accidental" degeneracy of the hydrogen spectrum with regard to the angular momentum quantum number ℓ arises from the conservation of the Runge-Lentz (or Laplace-Runge-Lentz-Pauli) vector

$$\boldsymbol{M} = \frac{\boldsymbol{p} \times \boldsymbol{L}}{\mu} - \frac{e^2 \boldsymbol{r}}{r}$$

Symmetry and the Discovery of Quarks

On the other hand, a degeneracy corresponds to a symmetry and a conserved quantity. The almost degenerate masses of the proton and neutron, e.g., can be interpreted in terms of an approximate symmetry under rotations in an abstract **isospin** space.

This approximate symmetry also explains the near-degeneracy of the charged and neutral pions, which form an $I = 1$ isospin triplet.



The discovery of the so-called "strange" particles $K^{\pm}, K^0, \ldots$ led to a proliferation of particle states with near-degenerate masses that could not be explained by isospin alone.

Symmetry and the Discovery of Quarks

In 1964, Gell-Mann showed that these could be explained by combining isospin and "strangeness" into a larger $\mathrm{SU}(3)$ symmetry based one three **Quarks** *u*, *d* and *s* as basic constituents.

Murray Gell-Mann
(1929–)

A simpler and more elegant scheme can be constructed if we allow non-integral values for the charges. We can dispense entirely with the basic baryon b if we assign to the triplet t the following properties: spin $\frac{1}{2}$, $z = -\frac{1}{3}$, and baryon number $\frac{1}{3}$. We then refer to the members $u^{\frac{2}{3}}$, $d^{-\frac{1}{3}}$, and $s^{-\frac{1}{3}}$ of the triplet as "quarks" 6) q and the members of the anti-triplet as anti-quarks $\bar{q}$. Baryons can now be constructed from quarks by using the combinations $(q\,q\,q)$, $(q\,q\,q\,q\,\bar{q})$, etc., while mesons are made out of $(q\,\bar{q})$, $(q\,q\,\bar{q}\,\bar{q})$, etc. It is assuming that the lowest baryon configuration $(q\,q\,q)$ gives just the representations **1**, **8**, and **10** that have been observed, while the lowest meson configuration $(q\,\bar{q})$ similarly gives just **1** and **8**.

6) James Joyce, Finnegan's Wake (Viking Press, New York, 1939) p.383.

Symmetry and the Discovery of Quarks

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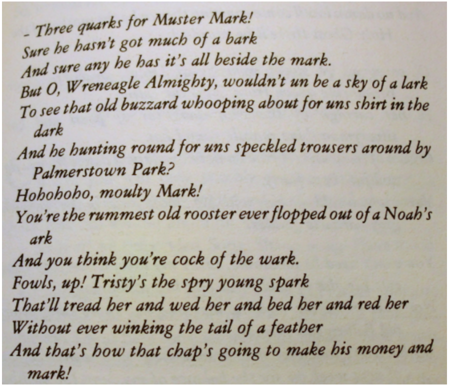
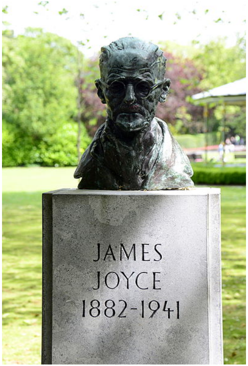


– Three quarks for Muster Mark!
Sure he hasn't got much of a bark
And sure any he has it's all beside the mark.
But O, Wreneagle Almighty, wouldn't un be a sky of a lark
To see that old buzzard whooping about for uns shirt in the dark
And he hunting round for uns speckled trousers around by Palmerstown Park?
Hohohoho, moulty Mark!
You're the rummest old rooster ever flopped out of a Noah's ark
And you think you're cock of the wark.
Fowls, up! Tristy's the spry young spark
That'll tread her and wed her and bed her and red her
Without ever winking the tail of a feather
And that's how that chap's going to make his money and mark!



Symmetry and the Discovery of Quarks

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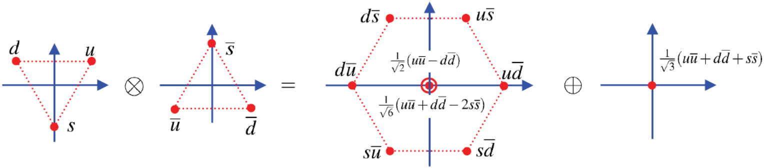
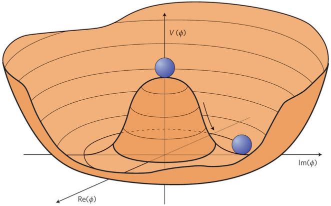


The rules of this "eightfold way" can be seen as a generalization of the rules of angular momentum addition and are an example of the application of **representation theory** in physics.

Spontaneous Symmetry Breaking

Physically particularly important is the case that the state of lowest energy does not share the full symmetry of the Lagrangian. In this case the symmetry is **spontaneously broken**.

An example from classical mechanics is given by a point particle in a "champagne bottle" potential.



An example from statistical physics is ferromagnetism where above the Curie temperature the spontaneous magnetization breaks isotropy in spite of the rotational invariance of the Hamiltonian.

Broken Symmetries and Massless Particles

Goldstone-Theorem: In a quantum field theory each spontaneously broken symmetry corresponds to a massless particle, a (Nambu-)Goldstone-boson.

JEFFREY GOLDSTONE
(1933–)

E.g. for massless quarks, the strong interactions breaks the chiral symmetry $\mathrm{SU}(N_f)_L \times \mathrm{SU}(N_f)_R \rightarrow \mathrm{SU}(N_f)_V$. The $(N_f^2 - 1)$ broken generators correspond to pseudoscalar Goldstone-bosons.

In the real world, the up- and down-quarks are very light, but not massless. Accordingly, the pions as pseudo-Goldstone bosons are light, but not exactly massless (**chiral perturbation theory**).

Local Symmetries

An especially important form of symmetries in field theory are local symmetries (**gauge symmetries**), under which the fields at different spacetime points can transform differently.

Global symmetry: $\phi(x) \mapsto f(\phi(x))$
Local symmetry: $\phi(x) \mapsto f(x, \phi(x))$

An example is classical electromagnetism, where the Maxwell equations are invariant under gauge transformations $A_\mu \mapsto A_\mu + \partial_\mu \chi$.

Local Symmetries and Gauge Theories

Gauge symmetries put very stringent constraints on the possible form of a field theory: there must be a gauge potential A_μ, which must occur in the combination $\partial_\mu + A_\mu$ in order to compensate the position dependence of the gauge transformations of the matter fields by its own transformation behavior.

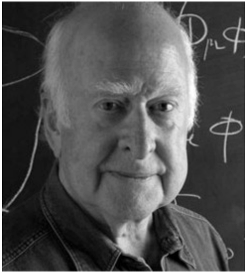
Gauge symmetry thus uniquely determines the form of the interaction.

The quanta of the gauge potential are necessarily massless **gauge bosons** of spin 1.

Examples of **gauge theories** are QED with gauge group $\mathrm{U}(1)$ and the photon as gauge boson, and QCD with gauge group $\mathrm{SU}(3)$ and the gluons as gauge bosons.

Spontaneous Breaking of Local Symmetries – Higgs-Effect

Higgs-Effect: The spontaneous breaking of a gauge symmetry does not give rise to Goldstone bosons; instead, the gauge bosons become massive.



Peter Higgs
(1929–)

The spontaneous breaking of electroweak symmetry $\mathrm{SU}(2)_L \times U(1)_Y \rightarrow U(1)_Q$ renders the gauge bosons $W^\pm$, Z^0 massive, while the photon corresponds to the unbroken gauge symmetry $U(1)_Q$ and remains massless.

The Higgs boson is the quantum of the remaining degree of freedom of the Higgs field whose non-vanishing expectation value breaks the symmetry.

Anomalies

Under certain circumstances, a symmetry of a classical field theory does not survive quantization. In this case, one speaks of an **anomaly** ($<$ gr. ἀ- "un-, not-", νόμος "law").

The global axial symmetry $\mathrm{U}(1)_A$, e.g., is anomalous in QCD and QED; this enables the decay $\pi^0 \to \gamma\gamma$ (and makes it possible to predict the decay rate).

An anomaly in a gauge symmetry, on the other hand, renders the theory inconsistent. In order for all possible anomalies to cancel in the Standard Model, the quarks must carry charges of $\frac{2}{3}$ and $-\frac{1}{3}$ of the electron charge, thus ensuring that the charges of the proton and electron are equal and opposite, and thus that atoms are neutral.

Overview of Symmetries

	global	local
exact	conservation laws, degenerate states	massless gauge bosons
spontaneously broken	massless Goldstone-bosons	massive vector bosons
anomalous	various effects	(*inconsistent*)

Course Outline

1. Elements of group theory
2. Elements of Lie group theory
3. Elements of representation theory
4. Some physical applications
5. The Poincaré group
6. Local symmetries
7. Broken symmetries

Course Outline

1. Elements of group theory
   - basic definitions
   - conjugacy classes and cosets
   - homomorphisms
2. Elements of Lie group theory
3. Elements of representation theory
4. Some physical applications
5. The Poincaré group
6. Local symmetries
7. Broken symmetries

Course Outline

1. Elements of group theory
2. Elements of Lie group theory
   - Lie groups and Lie algebras
   - basic properties
   - Cartan-Dynkin classification
3. Elements of representation theory
4. Some physical applications
5. The Poincaré group
6. Local symmetries
7. Broken symmetries

Course Outline

1. Elements of group theory
2. Elements of Lie group theory
3. Elements of representation theory
    - basic definitions
    - Maschke's theorem and Schur's lemmas
    - Clebsch-Gordan series
    - Young tableaux
    - highest-weight representations
4. Some physical applications
5. The Poincaré group
6. Local symmetries
7. Broken symmetries

Course Outline

1. Elements of group theory
2. Elements of Lie group theory
3. Elements of representation theory
4. Some physical applications
    - Noether's theorem
    - Wigner-Eckart theorem
    - Quark model
    - Coulomb problem
5. The Poincaré group
6. Local symmetries
7. Broken symmetries

Course Outline

1. Elements of group theory
2. Elements of Lie group theory
3. Elements of representation theory
4. Some physical applications
5. The Poincaré group
   - representations of the Lorentz group
   - representations of the Poincaré group
   - massive and massless particles, spin and helicity
6. Local symmetries
7. Broken symmetries

Course Outline

1. Elements of group theory
2. Elements of Lie group theory
3. Elements of representation theory
4. Some physical applications
5. The Poincaré group
6. Local symmetries
   - gauge transformations and gauge fields
   - gauge fixing, Fadeev-Popov procedure
   - BRST symmetry
7. Broken symmetries

Course Outline

1. Elements of group theory
2. Elements of Lie group theory
3. Elements of representation theory
4. Some physical applications
5. The Poincaré group
6. Local symmetries
7. Broken symmetries
    - Goldstone effect and χPT
    - Higgs effect and the Standard Model
    - Anomalies and consistency