Examples Sheet 1

Symmetries in Physics

Winter 2019/20

Lecturer: PD Dr. G. von Hippel

1. *Groups and their properties* (10 P.)

   Which of the following structures are groups? Which of the groups are abelian? Which groups are finite, and what is the order of each finite group?

   1. $\mathbb{R}$ with addition
   2. $\mathbb{R}$ with multiplication
   3. $\mathbb{R}\backslash\{0\}$ with multiplication
   4. $(0; \infty)$ with multiplication
   5. $\{0, 1\}$ with multiplication
   6. $\{1, -1\}$ with multiplication
   7. $\{0, \ldots, k-1\}$ with addition modulo k
   8. $\{\mathrm{e}^{2\pi ik/n} | k = 0, \ldots n-1\}$ with multiplication
   9. $\mathbb{R}^3$ with vector addition
   10. $\mathbb{R}^3$ with the vector product
   11. $\{M \in \mathbb{R}^{n\times n} | \det(M) = 0\}$ with matrix multiplication
   12. $\{M \in \mathbb{R}^{n\times n} | \det(M) = 1\}$ with matrix multipication
   13. $\{M \in \mathbb{R}^{n\times n} | \det(M) \neq 0\}$ with matrix multipication
   14. $\{M \in \mathbb{R}^{n\times n} | M^t = M\}$ with matrix multipication
   15. $\{M \in \mathbb{R}^{n\times n} | M^t = M^{-1}\}$ with matrix multipication

2. *Consequences of the group axioms* (10 P.)

   1. Let G be a group with neutral element e. Starting from the group axioms, show that
      (a) $\forall a \in G\ ae = a$
      (b) $\forall a \in G\ aa^{-1} = e$
      (c) $\forall a, b \in G\ (ab)^{-1} = b^{-1}a^{-1}$
   2. Show starting from the group axioms that the existence of the neutral element also implies its uniqueness.
   3. In the same manner, show the uniqueness of the inverse.

3. *Multiplication tables of small finite groups* (10 P.)

    1. Starting from the group axioms, show that the multiplication table of a finite group must be a *latin square*, i.e. that each element must occur exactly once in each row and column.
    2. Write down the multiplication tables of all groups of order n for $n = 1, 2, 3, 4$, and show that your list is complete.