# Theoretical Physics 5 : WS 2019/2020
# Exercise sheet 1

14.10.2019

## 1. Exercise (70 points) : Non-relativistic neutron and electron gases

At extremely high density inside stars, inverse beta decay, $e^- + p^+ \to n + \nu$, converts virtually all of the protons and electrons into neutrons. These stars (called neutron stars) are stabilised against gravitational collapse by the degeneracy pressure of their neutrons (this is a pressure of free neutron gas). Assuming constant density, the radius R of such an object can be calculated as follows:

**(a)(5 points)** Write the total neutron energy in terms of the radius R, the number of neutrons N and the neutron mass M.

**(b)(15 points)** Calculate the gravitational energy of a uniformly dense sphere. Express the result in terms of the gravitational constant $G$, $R$, the number of neutrons N and the neutron mass M.

**(c)(15 points)** Find the radius for which the total energy is minimal. Express this equilibrium radius R a function of $\hbar, q, G, M$, and $N$. Express R numerically as function of N.

**(d)(5 points)** Determine the radius of a neutron star with mass of the sun. (Sun mass - $1.989 \times 10^{30}$ kg, nucleon mass - $M = 1.674 \times 10^{-27}$ kg). Compare the radius of the neutron star with the radius of the Earth.

**(e)(10 points)** Determine the Fermi energy for the neutron star, in MeV, and compare it with rest energy of a neutron. Are neutrons relativistic ?

**(e)(20 points)** Other cold stars (called white dwarfs) are stabilised against gravitational collapse by the degeneracy pressure of their electrons (this is a pressure of free electron gas). Repeat (a)-(e) for white dwarfs, the number of electrons per nucleon q, where the electron mass m can be neglected in comparison with nucleon mass M, and where N is the number of nucleons (protons and neutrons). Assume $q = 1/2$ for the numerical evaluation. Are electrons relativistic ?

## 2. Exercise (30 points) : Exchange forces in a quantum well

Consider 2 particles in a one-dimensional infinite potential well

$$V(x) = \begin{cases} \infty & \text{when } x < 0 \\ 0 & \text{when } 0 \leq x \leq L \\ \infty & \text{when } x > L \end{cases}$$

with $L > 0$, and where the particles are non-relativistic.

**(a)(5 points)** Find the energy spectrum and normalized wave-functions for one particle. Classify the states as eigenstates of the parity operator. Is it possible to say *a priori* if these states are eigenstates of the parity operator ?

**(b)(8 points)** Evaluate $< x >_a$, $< x^2 >_a$ and $< x >_{ab}$. Find a condition on a and b for non-vanishing $< x >_{ab}$. Consider the case of $a = b$ separately. Is it possible to have a state of two fermions with $a = b$?

**(c)(7 points)** Evaluate $< (x_1 - x_2)^2 >$ for 2 particles 1,2 in a quantum well in states a,b in case of distinguishable particles, as well as for bosons, and fermions. Find numerically the relative influence of the exchange force for the eight lowest states of energy for both particles.

**(d)(10 points)** Consider particles with spin 1/2 in a quantum well.