

Exercise sheet 2  
Theoretical Physics 5 : WS 2019/2020  
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**Exercise 0.**

How much time did it take to complete the task?

**Exercise 1. (20 points) : Boson wave functions**

Write explicitly the following  $N$ -boson wave functions in terms of single-boson wave functions  $\psi_i(x_j)$

- a) (5 p.)  $\Phi_{3000\dots 0}(x_1, x_2, x_3);$
- b) (5 p.)  $\Phi_{1101\dots 0}(x_1, x_2, x_3);$
- c) (5 p.)  $\Phi_{0121\dots 0}(x_1, x_2, x_3, x_4);$
- d) (5 p.)  $\Phi_{3001\dots 0}(x_1, x_2, x_3, x_4);$

**Exercise 2. (30 Points) : Number of Bosons**

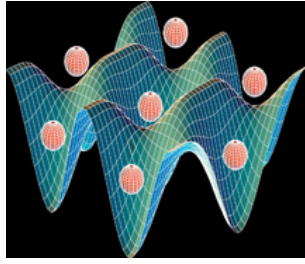
Consider the particle number operator  $N = \sum_i C_i^\dagger C_i$  for a system of bosons.

- a) (20 p.) Show that it commutes with the Hamiltonian

$$H = \sum_{i,j} \langle i|H_0|j\rangle C_i^\dagger C_j + \frac{1}{2} \sum_{i,j,k,l} \langle i,j|V|k,l\rangle C_i^\dagger C_j^\dagger C_k C_l.$$

What is the physical meaning of this commutation relation?

- b) (5 p.) Compute  $[N, (C_i)^n];$
- c) (5 p.) Compute  $[N, C_i C_j];$



### Exercise 3. (25 Points) : Bose-Hubbard model

The Bose-Hubbard model gives an approximate description of the physics of interacting bosons on a lattice. It can be used to study systems such as bosonic atoms on an optical lattice, *i.e.* a periodic trap formed by the interference of counterpropagating laser beams. This system resembles a crystal in the sense that the atoms are in a periodic potential. The Hamiltonian of this model is given by (Latin indices refer to lattice sites)

$$H = -t \sum_{\langle i,j \rangle} (C_i^\dagger C_j + C_j^\dagger C_i) + \frac{U}{2} \sum_i C_i^\dagger C_i (C_i^\dagger C_i - 1),$$

where  $\langle i, j \rangle$  means that the sum is restricted over first neighbors only, and  $U > 0$ .

- Provide an interpretation of each term of this Hamiltonian; (10 points)
- Show that in this model, the number of particles is conserved; (10 points)
- Discuss qualitatively the limits  $t \ll U$  and  $t \gg U$ . What kind of phenomenon can this model reproduce? (5 points)

### Exercise 4. (25 Points) : Bulk Properties of Copper

Let us assume that copper can be described as a non-relativistic free electron gas. The density of copper is  $8.96 \text{ gm/cm}^3$ , its atomic weight is  $63.5 \text{ gm/mole}$ , and the number of free electrons per copper atom is well-approximated by 1.

- (5 p.) Calculate the electron Fermi energy for copper in eV. Is it safe to assume that the electrons in copper are non-relativistic?
- (5 p.) Calculate the Fermi temperature for copper, namely the temperature at which the characteristic thermal energy ( $k_B T$ , where  $k_B$  is the Boltzmann constant and  $T$  is the Kelvin temperature) equals the Fermi energy for copper. Solid copper has a melting point of  $1356 \text{ K}$ . Is it safe to assume that the electrons in solid copper are close to the ground state configuration?
- (5 p.) Calculate the degeneracy pressure of copper.
- (10 p.) The bulk modulus of a material measures how it responds to uniform compression:

$$B = -V \frac{\partial P}{\partial V}$$

where  $V$  is the volume of the material and  $P$  is the pressure. What is  $B$  for a free electron gas? How well does the free electron gas model account for the bulk modulus of copper,  $13,4 \times 10^{10} \text{ N/m}^2$ ? Is this expected?