Exercise sheet 12
Theoretical Physics 6a (QFT): SS 2019

01.07.2019

Exercise 1. (50 points) : Self-energy in scalar QED

(a)(25 points) Calculate the self-energy graphs for a scalar particle in QED in dimensional regularization.

(b)(10 points) What are the mass and field strength counterterms in dimensional regularization in the $\overline{MS}$ scheme?

(c)(10 points) What is the final (finite) expression for the renormalized propagator at one-loop level? Work out the remaining Feynman parameter integral.

(d)(5 points) Express the above used $\overline{MS}$ mass in terms of the pole mass for the scalar propagator at one-loop level.

Exercise 1. (50 points) : β-function in QED

In the lecture notes we have calculated the effective (running) coupling in QED at 1-loop level as:

$$e_R^2(Q^2) = e_R^2 \left\{ 1 + \frac{e_R^2}{12\pi^2} \ln \frac{Q^2}{m^2} + \mathcal{O}(e_R^4) \right\}, \tag{1}$$

where e_R^2 on the *rhs* can be interpreted as the coupling defined at $Q^2 = m^2$. The β-function in QED expresses in general how the coupling constant

changes with mass scale Q as:

$$\beta(e_R) \equiv Q\frac{de_R}{dQ}. \tag{2}$$

(a)(10 points) Use Eq. (1) to express the β-function at leading order in terms of the renormalized coupling $e_R(Q^2)$.

(b)(10 points) Which power in e_R do the correction terms to the leading order result for the β-function have ?

(c)(10 points) Discuss from the sign of the β function how it behaves at high energies (short distances), corresponding with $Q \to \infty$.

(d)(20 points) By using the result obtained in (a) for the β-function, you obtain a differential equation for $e_R(Q^2)$. Solve this equation by expressing the running coupling $e_R(Q^2)$ at scale Q in terms of the running coupling $e_R(\mu^2)$ at an arbitrary scale μ. Show that when taking $\mu = m$, you find back the result of Eq. (1) up to correction terms of $\mathcal{O}(e_R^6)$.