Effective Field Theory

EFTs of the SM **Series 4**

Assignment 1:

Show the Gordon-identity

$$\overline{u}(p_f)\gamma^\mu u(p_i) = \frac{1}{2m}\overline{u}(p_f)\Big\{(p_f+p_i)^\mu + i\sigma^{\mu\nu}(p_f-p_i)_\nu\Big\}u(p_i), \tag{1}$$

for on shell particles, i.e.

$$\not{p}u(p) = \gamma^\mu p_\mu u(p) = mu(p), \tag{2}$$

$$\overline{u}(p)\not{p} = \overline{u}(p)m, \tag{3}$$

with

$$\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]. \tag{4}$$

Hint:

$$\gamma^\mu \not{p} = 2p^\mu - \not{p}\gamma^\mu. \tag{5}$$

Assignment 2:

For products of quark bilinears Fierz identities prove very useful, which allow for a rearrangement of the spinor fields in expressions as

$$(\overline{u}_1\Gamma^A u_2)(\overline{u}_3\Gamma^B u_4). \tag{6}$$

The Dirac matrices can be decomposed in a complete set of 16 matrices

$$\Gamma^A \in \{\mathbb{1}, \gamma_5, \gamma_\mu, \gamma_\mu\gamma_5, \sigma_{\mu\nu}\}. \tag{7}$$

(a) Normalize the 16 matrices Γ^A to the convention (ignoring signs from $g_{\mu\nu}$)

$$\mathrm{Tr}\Big[\Gamma^A\Gamma^B\Big] = 4\delta^{AB}. \tag{8}$$

Hint: Use the fact that the trace is cyclic

$$\mathrm{Tr}[\gamma^\mu\gamma^\nu] = \mathrm{Tr}[\gamma^\nu\gamma^\mu]. \tag{9}$$

Note that γ_5 is traceless and use $\mathrm{Tr}[\mathbb{1}] = 4$ in 4 dimensions.

(b) Write the general Fierz identity as an equation

$$(\overline{u}_1\Gamma^A u_2)(\overline{u}_3\Gamma^B u_4) = \sum_{C,D} C^{AB}_{CD}(\overline{u}_1\Gamma^C u_4)(\overline{u}_3\Gamma^D u_2) \tag{10}$$

with unknown coefficients C^{AB}_{CD}. Use the completeness of the Γ^A, where any matrix X can be written

$$X = C^A \Gamma^A, \quad C^A = \frac{1}{4}\mathrm{Tr}\Big[X\Gamma^A\Big] \tag{11}$$

leading to

$$\delta_{nj}\delta_{km} = \sum_C \frac{1}{4}(\Gamma^C)_{nm}(\Gamma^C)_{kj} \tag{12}$$

to show that

$$C^{AB}_{CD} = \frac{1}{16}\mathrm{Tr}\Big[\Gamma^C\Gamma^A\Gamma^D\Gamma^B\Big]. \tag{13}$$

Hint: It is instructive to write out the spinor products in components, i.e.

$$\overline{u}_i\Gamma^A u_j = (\Gamma^A)_{ij} \tag{14}$$

(c) Work out explicitly the Fierz identity for

$$(\overline{u}_1\gamma^\mu u_2)(\overline{u}_3\gamma^\mu u_4). \tag{15}$$

Assignment 3 (Bonus):

Derive the Feynman rules in momentum space for the interaction terms of the Euler-Heisenberg-Lagrangian

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}_{\mu\nu} + \frac{c_1}{m^4}\Big(F_{\mu\nu}F^{\mu\nu}\Big)^2 + \frac{c_2}{m^4}F_{\mu\nu}F^{\nu\rho}F_{\rho\sigma}F^{\sigma\mu}. \tag{16}$$