Lattice QCD activities on dark matter production and detection

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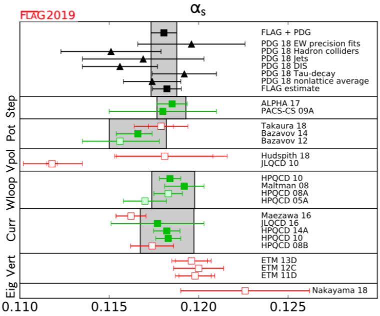
CERN, 14 March 2019

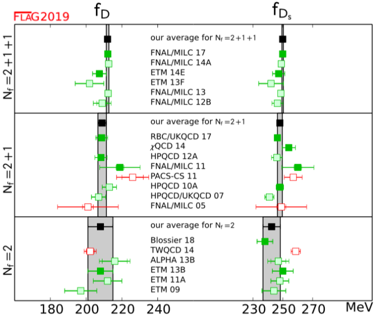
Cluster of Excellence

Lattice QCD is a regularization of QCD. . .

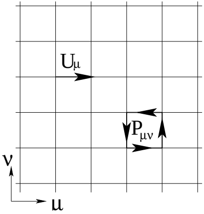
- maintaining an exact form of SU(3) gauge symmetry,
- using compact gauge variables ($\rightsquigarrow$ no gauge fixing required),
- breaking continuous space-time symmetries,
- formulated in the Euclidean-space path integral representation,
- well-suited for non-perturbative treatment via importance-sampling simulation techniques.

A good place to find the state-of-the-art as well as pointers to the literature: the latest FLAG report 1902.08191. Examples:





Space-time as a four-dimensional lattice



Gluons: $U_\mu(x) = e^{iag_0 A_\mu(x)} \in SU(3)$
'link variables'

Quarks: $\psi(x)$ 'on site', Grassmann;
discretized Dirac operator D[U]

Imaginary-time path-integral:

$$Z_{\rm QCD} = \int DU\, D\bar\psi\, D\psi\; e^{-S_E[U,\bar\psi,\psi]} = \int DU \det(D[U]+m) e^{-S_{\rm gauge}[U]}$$

- Quark correlation functions are expressed in terms of quark propagators; however, these propagators must be computed in a non-perturbative SU(3) gauge field background.
- $\Rightarrow$ Most of the computing time goes into solving the discretized Dirac equation in a background field.
- Representative samples of the gauge fields are generated, QCD expectation values obtained as averages over these configurations.
- Recently, idea of the master field [M. Lüscher]: with very large lattices, "more volume= more statistics".

Outline

- Nucleon structure for neutrino and WIMP detection
- Thermal QCD spectral functions for the production of sterile neutrinos
- Axions: the topological susceptibility in the quark-gluon plasma phase.

Nucleon structure for neutrino and WIMP detection

I will discuss

- isovector axial form factor: relevant for neutrino detection
- scalar matrix elements: relevant for the direct detection of WIMPs.

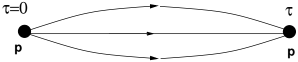
Calculating the Nucleon Mass on the Lattice

Use an interpolating field such as

$$\chi(x) = \epsilon^{abc}\left(u^{Ta}(x) C\gamma_5 d^b(x)\right)\, u^c(x)$$

Two-point function:

$$C_2(\tau,\boldsymbol{p}) = \Gamma^{\beta\alpha}\, a^3 \sum_{\boldsymbol{x}} e^{i\boldsymbol{p}\cdot\boldsymbol{x}}\, \langle 0|\chi^\alpha(x)\bar{\chi}^\beta(0)|0\rangle$$



quark action quadratic $\Rightarrow$ Wick contractions,
$\langle\psi(y)\bar{\psi}(x)\rangle = (D[U])^{-1}(y,x)$

Spectral representation: $\langle 0|\chi(0)|N,p,s\rangle = Z_N\sqrt{\frac{M}{E_p}}\,u(p,s)$

$$C_2(\tau,\boldsymbol{p}) \overset{\tau\to\infty}{=} \frac{Z_N^2}{2E_p}\, e^{-E_p\tau}\, \mathrm{Tr}\left\{\Gamma(-i\not{p}+M)\right\} + \dots$$

Nucleon form factor calculations

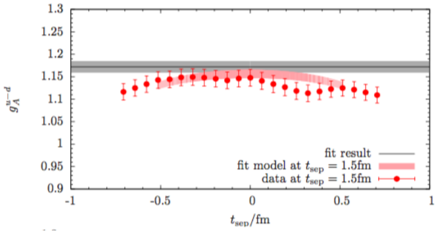
Let Ψ be an interpolating field for the nucleon, $\langle N|\Psi|\text{vac}\rangle \neq 0$:

$$C_2(t,\boldsymbol{p}) = \sum_{\boldsymbol{x}} e^{i\boldsymbol{p}\cdot\boldsymbol{x}}\Gamma_{\beta\alpha}\,\langle \Psi_\alpha(t,\boldsymbol{x})\Psi_\beta(0)\rangle$$

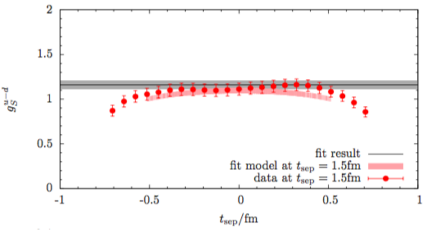
$$C_3(t,t_s,\boldsymbol{q}) = \sum_{\boldsymbol{x},\boldsymbol{y}} e^{i\boldsymbol{q}\cdot\boldsymbol{y}}\Gamma_{\beta\alpha}\,\langle \Psi_\alpha(t_s,\boldsymbol{x})J(t,\boldsymbol{y})\Psi_\beta(0)\rangle$$

Calculation of charges ($\boldsymbol{q}=0$): form the ratio
$R(t,t_s,\mathbf{0}) = C_3(t,t_s,\mathbf{0})/C_2(t_s,\mathbf{0})$ for $t \simeq t_s/2 \longrightarrow \infty$.

g_A^{u-d}

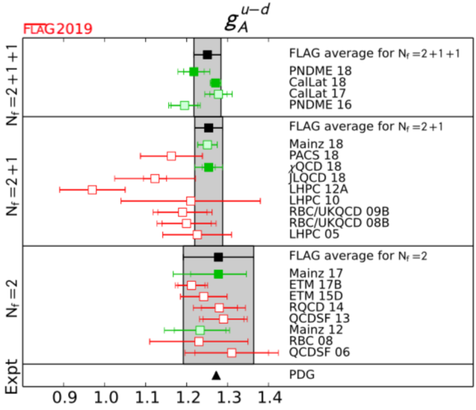


g_S^{u-d}



K. Ottnad et al. (Mainz lattice group) 1809.10638

Status: the isovector axial charge of the nucleon



- FLAG lattice average for $N_{\rm f} = 2 + 1$ QCD: $g_A^{u-d} = 1.254(0.016)(0.030)$
  J. Liang et al, 1806.08366 (PRD).
- PDG average: $g_A^{u-d} = 1.2724 \pm 0.0023$

The scalar matrix elements of the nucleon

$$\begin{aligned} \sigma_{\pi N} &= m_{ud}\,\langle N|\bar{u}u + \bar{d}d|N\rangle \\ \sigma_s &= m_s\,\langle N|\bar{s}s|N\rangle \\ \sigma_c &= m_c\,\langle N|\bar{c}c|N\rangle. \end{aligned}$$

Determine the spin-independent interaction of WIMPs with the nucleon.

Due to the heavy mass of the WIMP, the momentum transfer is very small $\Rightarrow$ only need to know the forward matrix element.

Technically, the matrix elements above require the calculation of quark-disconnected diagrams, which are computationally very demanding.

NB. There are recent lattice calculations of axial, scalar and tensor matrix elements in light nuclei at heavy quark masses $(m_\pi = 800\,\mathrm{MeV})$. Observation: the nuclear interactions/correlations can affect the charge up to the $\sim 10\%$ level. [NPLQCD lattice collaboration, 1712.03221 (PRL)]

Results for the sigma-terms

Light quarks: FLAG average for $N_\mathrm{f} = 2+1$ QCD:

$$\sigma_{\pi N} = 39.7(3.6)\mathrm{MeV}$$

Most recent dispersive analysis: Hoferichter, de Elvira, Kubis, Meissner, 1506.04142 (PRL)

$$\sigma_{\pi N} = (59.1 \pm 3.5)\mathrm{MeV}.$$

This represents a tension of 3.9 standard deviations.

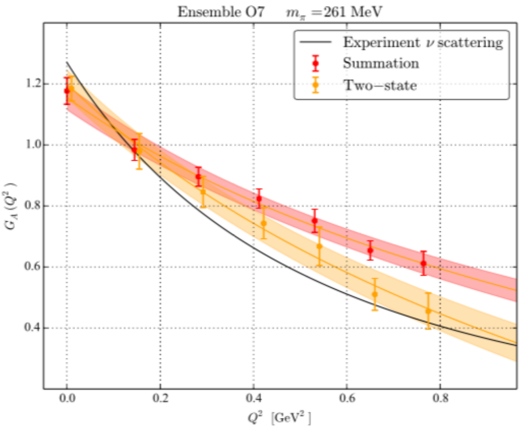
Strangeness:

$$\sigma_s = 52.9(7.0)\mathrm{MeV}$$

Charm: Using the heavy-quark expansion, [Hill & Solon, 1409.8290 (PRD)]

$$\sigma_c = 68.5 \pm 2.8\mathrm{MeV}.$$

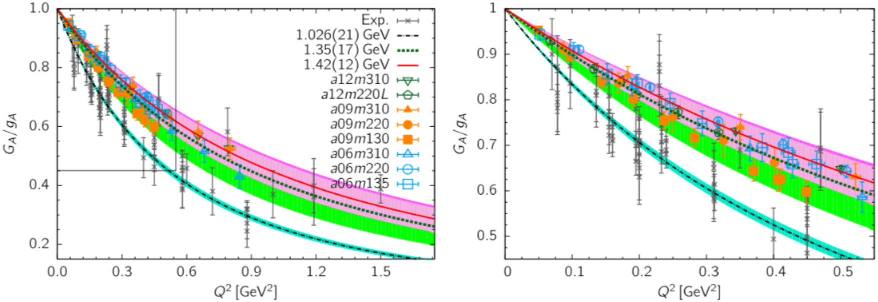
Axial form factor of the nucleon



- example of a calculation on an $N_\mathrm{f} = 2$ $64^3 \times 128$ lattice, $m_\pi \simeq 260\,\mathrm{MeV}$, $a = 0.050\,\mathrm{fm}$;
- the systematics of extracting the ground-state matrix element is not negligible;
- $r_A = 0.60(7)\,\mathrm{fm}$; cf. pheno. dipole fit: $r_A^{\mathrm{pheno}} = 0.666(14)\,\mathrm{fm}$.

Mainz lattice group 1705.06186, (Int.J.Mod.Phys)

Axial form factor of the nucleon (2)



- turquoise band = dipole fit to exp^t data with dipole mass $M_A = 1.026(21)$ GeV;
- the green band represents the more recent MiniBoone estimate (from carbon data), $M_A = 1.35(17)$ GeV;
- the form factor calculated on the lattice is more in line with the latter: $r_A = 0.48(4)$ fm or $M_A = 1.42(12)$ GeV.

PNDME collaboration, 1705.06834 (PRD).

Production of keV-mass sterile neutrinos in the early universe: $T \sim 200$ MeV

The differential production rate of sterile neutrinos per unit volume is a linear combination of

$$\begin{aligned} \mathrm{Im}\,\not{\Sigma}_{\alpha\alpha}(q^0,\mathbf{q}) &= 4G_F^2 \sum_{H=W,Z} p_H \int \frac{\mathrm{d}^3\mathbf{r}}{(2\pi)^3} \frac{\cosh(\beta q^0/2)}{4E_1\cosh(\beta E_1/2)} \times \\ &\times \left[\frac{\gamma^\mu \Delta(-E_1,\mathbf{q}+\mathbf{r},-m_{l_H})\gamma^\nu}{\sinh[\beta(q^0+E_1)/2]} \rho^H_{\mu\nu}(-q^0-E_1,\mathbf{r}) - (E_1 \to -E_1) \right]. \end{aligned}$$

$$p_W = 2, \quad p_Z = 1/2, \quad \Delta(P,m) = \not{P} + m, \qquad E_1 = \sqrt{(\boldsymbol{q}+\boldsymbol{r})^2 + m_{l_H}^2}$$

$$m_{l_W} = m_{l_\alpha} = \text{mass of the charged lepton of generation } \alpha$$

$$m_{l_Z} = m_{\nu_\alpha} \simeq 0 = \text{mass of the MSM active neutrino}$$

$$2\mathrm{Im}\Sigma^0 = \Gamma_\nu = \text{the active neutrino damping rate}$$

$\rho^H_{\mu\nu}$ are QCD thermal spectral functions. Assuming SU(3)$_{\text{flavor}}$ symmetry:

$$\rho^H_{\mu\nu} = \frac{1}{4}(|V_{\text{ud}}|^2 + |V_{\text{us}}|^2)\left(\rho^{V,8}_{\mu\nu} + \rho^{A,8}_{\mu\nu}\right)$$

$$\rho^Z_{\mu\nu} = \frac{2}{3}\left[(1-2\sin^2\theta_W)^2\rho^{V,8}_{\mu\nu} + \rho^{A,8}_{\mu\nu}\right] + \frac{1}{36}\left[\rho^{V,0}_{\mu\nu} + \rho^{A,0}_{\mu\nu}\right].$$

Asaka, Laine, Shaposhnikov hep-ph/0605209 (JHEP).

Production of photons

The differential photon rate per unit volume of quark-gluon plasma:

$$d\Gamma(\boldsymbol{k}) = e^2 \; \frac{d^3k}{(2\pi)^3\, 2k} \; \frac{-\rho^{\mu}{}_{\mu}(\omega, \boldsymbol{k})}{e^{\beta k} - 1}$$

- $\rho_{\mu\nu}(\omega, \boldsymbol{k})$ is the thermal spectral function of the electromagnetic current ($= \rho_{\mu\nu}^{V,8}$ in the notation of the previous slide).
- this rate is measured in heavy-ion collisions, integrated over the space-time history of the 'fireball', assuming local thermal equilibrium is reached [see e.g. Paquet et al, 1509.06738 (PRC)].
- can we first test our ability to calculate $d\Gamma(\boldsymbol{k})$ on the lattice?

Ghiglieri et al. 1604.07544; Brandt et al. 1710.07050.

Definitions

Euclidean-time vector correlators ($\{\gamma^\mu,\gamma^\nu\} = 2g^{\mu\nu} = 2\mathrm{diag}(1,-1,-1,-1)$),

$$G^{\mu\nu}(x_0,\boldsymbol{k}) = \int d^3x\; e^{-i\boldsymbol{k}\cdot\boldsymbol{x}} \Big\langle j^\mu(x)\, j^\nu(y)\Big\rangle, \qquad j^\mu = \sum_f Q_f\, \bar\psi_f \gamma^\mu \psi_f$$

- all diagonal components of $G^{\mu\nu}$ are positive; spectral representation:

$$G^{\mu\nu}(x_0,\boldsymbol{k}) \overset{\mu=\nu}{=} \int_0^\infty \frac{d\omega}{2\pi}\,\rho^{\mu\nu}(\omega,\boldsymbol{k})\,\frac{\cosh[\omega(\beta/2-x_0)]}{\sinh(\beta\omega/2)}.$$

- from current conservation: $\omega^2\rho^{00}(\omega,k) = k^i k^j \rho^{ij}(\omega,k)$.
- consider the linear combination

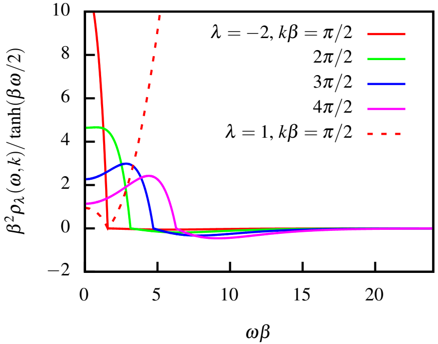
$$\rho(\omega,k,\lambda) \quad = \quad (\delta^{ij} - \hat{k}^i\hat{k}^j)\rho^{ij} + \lambda\,(\hat{k}^i\hat{k}^j\rho^{ij} - \rho^{00}) \qquad k \equiv |\boldsymbol{k}|, \quad \hat{k}^i = k^i/k,$$

- The differential photon rate per unit volume of plasma:

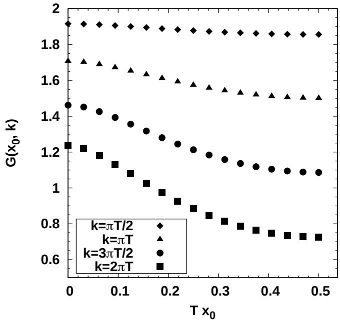
$$d\Gamma_\lambda(\boldsymbol{k}) = e^2\,\frac{d^3k}{(2\pi)^3\,2k}\,\frac{\rho(k,k,\lambda)}{e^{\beta k}-1} \quad \text{is independent of } \lambda.$$

The case of non-interacting fermions

Spectral function



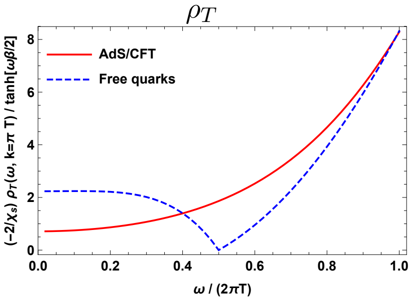
Euclidean correlator with $\lambda = -2$

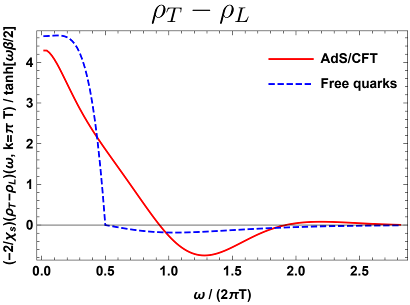


- determining the spectral function from the Eucl. correlator is a numerically ill-posed problem.

Qualitative form of the spectral functions: weak and strong coupling

Spatial momentum $k = \pi T$:

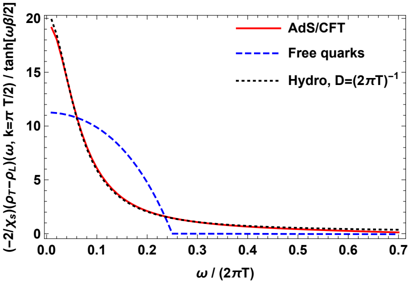




Spatial momentum $k = \pi T/2$:
At strong coupling, hydro works:

$$-2(\rho_T - \rho_L)(\omega, k)/\omega \approx \frac{4\chi_s\, Dk^2}{\omega^2 + (Dk^2)^2},$$

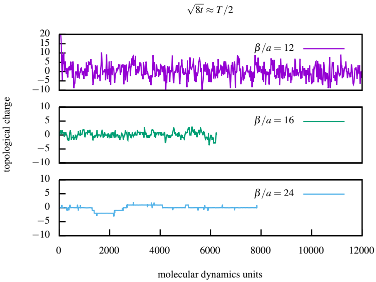
Refs: hep-th/0607237 and 1310.0164.

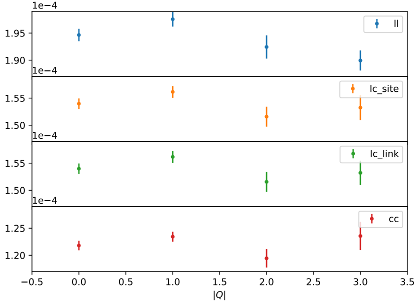


Lattice set-up with $N_f = 2$ $O(a)$-improved Wilson fermions

T (MeV)	T/T_c	β_{LAT}	β/a	L/a	$m_{\overline{MS}(2\,GeV)}$ (MeV)	N_{meas}
250	1.2	5.3	12	48	12	8256
"	"	5.5	16	64	"	4880
"	"	5.83	24	96	"	9600
500	2.4	6.04	16	64	"	8064

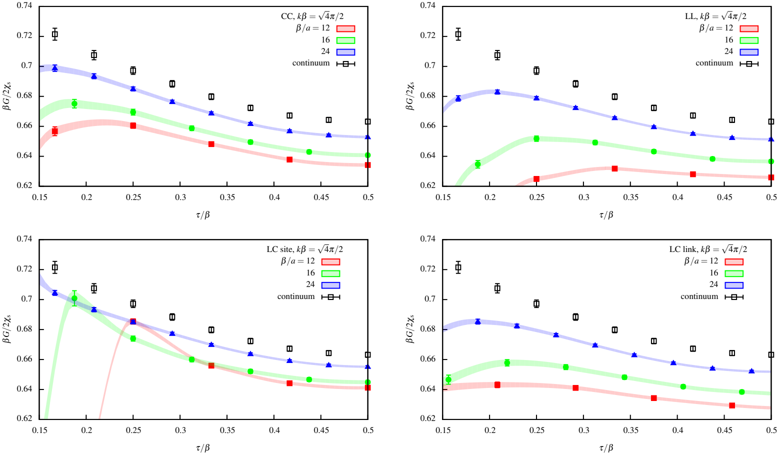
- enables continuum limit at $T = 250$ MeV





- only weak dependence of observable on topological charge
- impact of long autocorrelation time on vector correlator under control.

Continuum limit using tree-level improvement: $k = \pi T$



- Coarsest ensemble $N_t = 12$ is not included in the continuum extrapolation.
- In the subsequent analysis, we use the continuum-extrapolated correlator at $x_0 \geq \beta/4$.

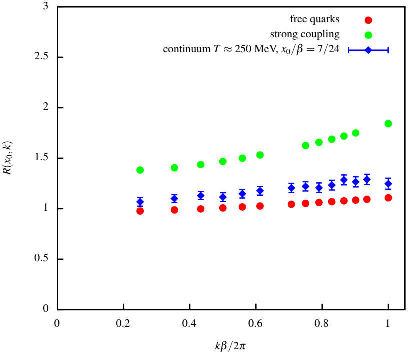
Can the lattice distinguish a weak- from a strong-coupling $\rho(\omega, k)$?

In the "transverse minus longitudinal" channel, consider the ratio

$$R(x_0, k) \equiv \frac{16\pi}{(\beta - 2x_0)^2 k^2} \left[\frac{G(x_0, k)}{G(\beta/2, k)} - 1\right]$$

This observable differs by a factor ~ 1.5 between the extreme cases of AdS/CFT and non-interacting quarks.

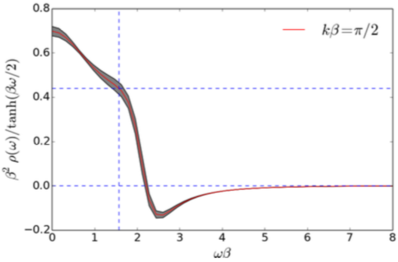
The continuum-extrapolated data lies between AdS/CFT and the free-quark prediction. Can the $\mathrm{O}(\alpha_s)$ corrections account for the lattice data?

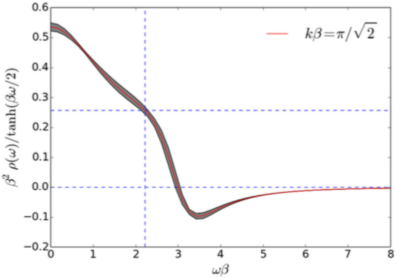


Estimating the spectral function

Fit ansatz: (consistent with known theory constraints; impose exact sum rule $\int_0^\infty d\omega\, \omega\rho(\omega,k) = 0$)

$$\frac{\rho(\omega,k)}{\tanh[\omega\beta/2]} = \frac{A(1+B\omega^2)}{[(\omega-\omega_0)^2+b^2][(\omega+\omega_0)^2+b^2][\omega^2+a^2]},$$





The lattice data is well described by a spectral function with a large photon production rate.

Talk by A. Toniato at this week's Theory workshop

Axions as dark matter candidate

Assume the axions form all of dark matter, and the phase of the Peccei-Quinn field ϕ starts out spatially random on distance scales larger than H^{-1}: (post-inflationary scenario)

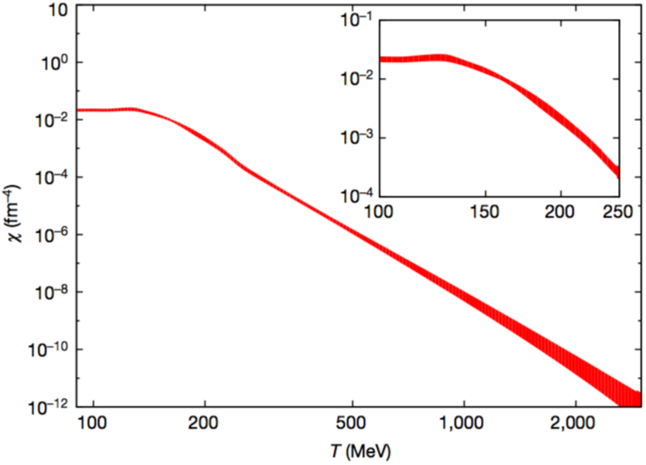
- the larger f_a, the more axions are produced $\Rightarrow$ constrain the possible values of f_a,
- and hence the axion mass: $m_a^2 = \chi_t / f_a^2$, where χ_t is the topological susceptibility of QCD at vanishing temperature.
- To calculate the relation between the final axion matter density and f_a, the thermal QCD topological susceptibility $\chi_t(T)$ is needed for $540 < T/\mathrm{MeV} < 1150$.
- Using lattice result for $\chi_t(T)$ on the next slide, the result of a sophisticated classical field theory calculation yields $m_a = 26.2 \pm 3.4\,\mu$eV.

1709.09466 G.D. Moore

Currently most comprehensive lattice results on $\chi_t(T)$

Difficulties:

- simulations can get stuck in fixed topological sectors;
- the topological susceptibility becomes extremely small at high T, $\sim T^{-8.6}$



Borsanyi et al, doi:10.1038/nature20115

Conclusion

- There is a significant number of activities in the lattice community related to the theme of dark matter.
- In several cases, the required QCD quantity cannot realistically be obtained from experimental information,
- but cross-checks with experiment for related quantities that can be measured are made.

A sum rule for $\rho \equiv \rho_{\lambda=-2}$

i. Lorentz invariance and transversity $\Rightarrow \tilde{G}_{\rm E}(\omega_n, k) = 0$ in vacuum and UV finite at $T > 0$

ii. OPE: from power-counting one expects $\tilde{G}_{\rm E}(\omega_n, k) \sim \frac{\langle \mathcal{O}_4 \rangle}{\omega_n^2}$

Furthermore, charge conservation demands $\tilde{G}_{\rm E}(\omega_n, k) \to 0$ as $k \to 0$ and $\omega_n \neq 0$, so actually

$$\tilde{G}_{\rm E}(\omega_n, k) \sim \frac{k^2 \langle \mathcal{O}_4 \rangle}{\omega_n^4}$$

iii. From the dispersive representation:

$$\tilde{G}_{\rm E}(\omega_n, k) = \int_0^\infty \frac{d\omega}{\pi} \omega \frac{\rho(\omega, k)}{\omega^2 + \omega_n^2} \xrightarrow{\omega_n \to \infty} \frac{1}{\pi \omega_n^2} \int_0^\infty d\omega\, \omega\, \rho(\omega, \mathrm{k})$$

The two expressions are only compatible if the super-convergent sum rule

$$\boxed{\int_0^\infty d\omega\, \omega \rho(\omega, \mathrm{k}) = 0}$$

holds.