The photon emission of quark-gluon plasma from lattice QCD

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Photon rate and spectral function

The photon rate

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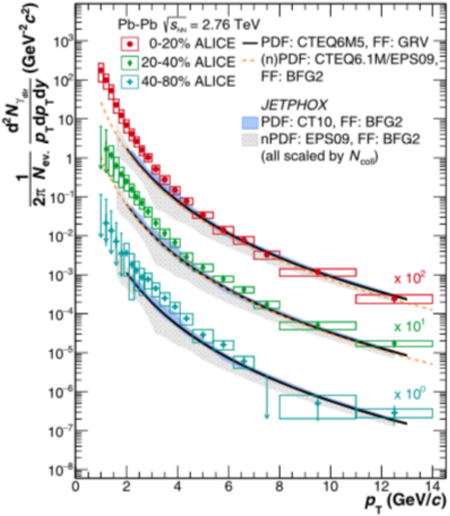
Direct photon spectrum in Pb-Pb collisions (not originating from hadron decays)

- Prompt direct photons, produced in hard scattering of partons, dominate at large p_T
- Thermal direct photons, created at the initial stage of the collision, dominate at low p_T. They carry information on the temperature, collective behavior and time evolution of the quark-gluon plasma



The thermal photon rate

- Vector current correlator:

$$G^{\mu\nu}(t,\vec{k}) = \int \mathrm{d}^3x\; e^{-i\vec{k}\cdot\vec{x}} \langle j^\mu(t,\vec{x}) j^\nu(0,\vec{y})\rangle$$

$$j^\mu = \sum_f Q_f\; \bar{\psi}_f \gamma^\mu \psi_f$$

- Spectral representation:

$$G^{\mu\nu}(t,\vec{k}) = \int_0^\infty \frac{\mathrm{d}\omega}{2\pi}\, \rho^{\mu\nu}(\omega,\vec{k})\, \frac{\cosh[\omega(\beta/2-t)]}{\sinh(\omega\beta/2)}\;, \qquad \beta = \frac{1}{T}$$

- Differential photon emission rate per unit volume:

$$\mathrm{d}\Gamma(\vec{k}) = e^2 \frac{\mathrm{d}^3k}{(2\pi)^3 2k} \frac{-\rho^\mu{}_\mu(k,k)}{e^{\beta k}-1}\;, \qquad k = \left|\vec{k}\right|$$

- Define the linear combination:

$$\rho(\omega,\vec{k},\lambda)=(\delta^{ij}-\hat{k}^i\hat{k}^j)\rho^{ij}+\lambda(\hat{k}^i\hat{k}^j\rho^{ij}-\rho^{00})\,,\qquad \hat{k}^i=k^i/k$$

for example: $\rho(\omega,k,1)=\rho^{ii}(\omega,k)-\rho^{00}(\omega,k)=-\rho^{\mu}{}_{\mu}(\omega,k)$

- The photon rate can be defined in terms of $\rho(\omega,k,\lambda)$:

$$\mathrm{d}\Gamma_\lambda(\vec{k})=e^2\frac{\mathrm{d}^3k}{(2\pi)^3 2k}\frac{\rho(k,k,\lambda)}{e^{\beta k}-1}$$

this expression is independent of λ, due to current conservation:

$$\omega^2\rho^{00}(\omega,k)=k^ik^j\rho^{ij}(\omega,k)$$

From now on we focus on $\underline{\lambda = -2}$.

$$\rho \equiv \rho(\omega, k, \lambda = -2) = (\delta^{ij} - 3\hat{k}^i\hat{k}^j)\rho^{ij}(\omega, k) + 2\rho^{00}(\omega, k)$$

Properties

- Non-negative for $\omega \leq k$
- In vacuum, Lorentz invariance and transversity of $G^{\mu\nu}(t, \vec{k})$ imply: $\boxed{\rho_{\lambda=-2}\,|_{\mathrm{vac}} = 0}$
  At $T > 0$ no new UV divergences appear $\Rightarrow$ ρ is UV-finite at $T > 0$
- OPE for the Euclidean correlator in momentum space (for $\lambda = -2$):
  - Power counting: $\tilde{G}(\omega_n, k) \underset{\omega_n \to \infty}{\sim} \langle \mathcal{O}_4 \rangle / \omega_n^2 + \ldots$
  - Charge conservation: $\tilde{G}(\omega_n, k) \underset{k \to 0}{\to} 0$, for $\omega_n \neq 0$

$$\boxed{\tilde{G}(\omega_n, k) \underset{\omega_n \to \infty}{\sim} k^2 \langle \mathcal{O}_4 \rangle / \omega_n^4}$$

Sum rule

The expansion of the dispersive representation

$$\tilde{G}(\omega_n, k) = \int_0^\infty \frac{\mathrm{d}\omega}{\pi} \omega \frac{\rho(\omega, k)}{\omega^2 + \omega_n^2} \quad \underset{\omega_n \to \infty}{\longrightarrow} \quad \frac{1}{\pi \omega_n^2} \int_0^\infty \mathrm{d}\omega \, \omega \rho(\omega, k) + O(\omega_n^{-4})$$

combined with the OPE, implies the sum rule:

$$\boxed{\int_0^\infty \mathrm{d}\omega \, \omega \rho(\omega, k) = 0}$$

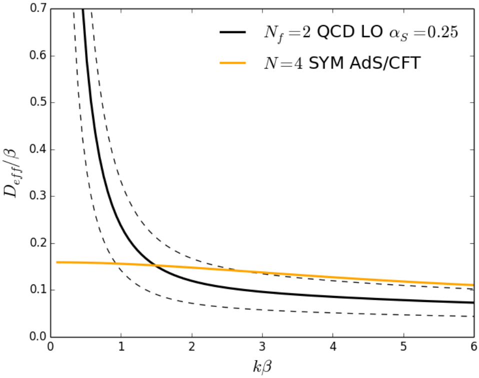
Effective diffusion coefficient

We compute the effective diffusion coefficient (proportional to the photon rate):

$$D_{eff}(k) = \frac{\rho(k,k)}{4k\chi_S}\,, \qquad \chi_S = \int \mathrm{d}^4x \langle j^0(x) j^0(0)\rangle$$

Perturbative QCD: hep-ph/0111107 (JHEP)

AdS/CFT: hep-ph/0607237



Lattice setup

- $N_f = 2$, $O(a)$-improved Wilson fermions

T (MeV)	T/T_c	β_{LAT}	β/a	L/a	$m_{\overline{\text{MS}}(2\,\text{GeV})}$ (MeV)	N_{meas}
250	1.2	5.3	12	48	12	8256
"	"	5.5	16	64	"	4880
"	"	5.83	24	96	"	9600
500	2.4	6.04	16	64	"	8064

- Continuum limit at $T = 250$ MeV
- Four independent discretizations of the isovector vector correlator $G^{\lambda=-2}(t, \vec{k})$ are considered
  - local or exactly-conserved lattice vector current
  - in the local-conserved case, two different definitions: conserved current defined on the site or in the midpoint of the link
- Projection to all spatial momenta, on- and off-axis, such that $k\beta \leq 2\pi$

Continuum limit

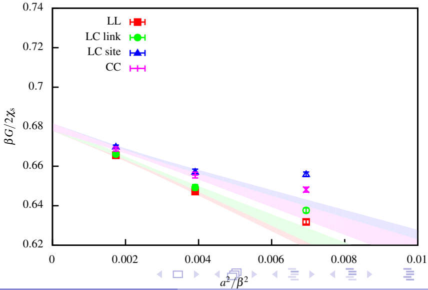
- Tree-level improvement of the correlator:

$$G^{\lambda=-2}(t,\vec{k}) \to \frac{G^{\lambda=-2}_{cont.t.l}(t,\vec{k})}{G^{\lambda=-2}_{lat.t.l}(t,\vec{k})} G^{\lambda=-2}(t,\vec{k})$$

- A piecewise spline interpolation of the correlators is performed before taking the combined continuum limit of the four discretizations

  - The coarsest ensemble $\beta/a = 12$ is not included in the continuum extrapolation
  - In the subsequent analysis we use the continuum-extrapolated correlator with $t \geq \beta/4$

$k\beta = \pi,\ t = \beta/3$



Fits to Padé ansatz

Padé ansatz for the spectral function

$$\frac{\rho(\omega,k)}{\tanh(\omega\beta/2)} = \frac{A(1+B\omega^2)}{(\omega^2+a^2)[(\omega+\omega_0)^2+b^2][(\omega-\omega_0)^2+b^2]}$$

- $\rho(\omega,k) \underset{\omega\to\infty}{\sim} 1/\omega^4$, consistent with OPE
- At small k, expect $a \sim Dk^2$ and $\omega_0, b \sim \mathrm{O}(T)$
- At every fixed k, 4-parameter fit:
  - scan in the non-linear parameters (a, b, ω_0)
  - the value of B is fixed by imposing the sum rule $\Rightarrow B = B(a, b, \omega_0)$
  - the value of A is fixed by χ^2 minimization
- It turns out that the χ^2 has a flat valley $\Rightarrow$ no strong constraints on the shape of the spectral function and on the value of the photon emission rate

Strategy for the global fit (1)

One may try to fit simultaneously data with different momenta, hoping to find stronger constraints

Polynomial ansatz for the k-dependence of the nonlinear parameters

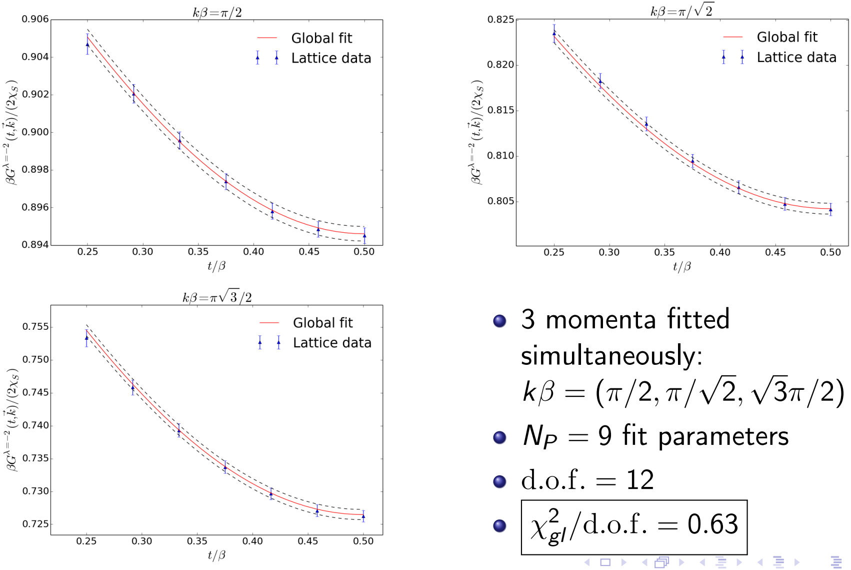
$$a(k) = a_0 + a_2 k^2 \,, \quad b(k) = b_0 + b_2 k^2 \,, \quad \omega_0(k) = W_0 + W_2 k^2$$

- N_k momentum values are included in the fit
- Scan in the non-linear parameters $(a_0, a_2, b_0, b_2, W_0, W_2)$
- At each k, B is determined by imposing the sum rule $\Rightarrow$ $B = B(a_0, a_2, b_0, b_2, W_0, W_2; k)$

Strategy for the global fit (2)

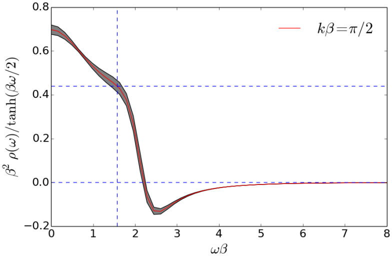
- N_k different values of A are determined by χ^2 minimization, at fixed k
  - Taking into account the correlation between data at different Euclidean time
  - But multiplying by $x = 0.85$ the off-diagonal elements of the covariance matrix
  - The regularization is necessary not because the covariance matrix is poorly determined, but because the most accurate modes still suffer from cutoff effects
- Global χ^2: $\boxed{\chi^2_{gl} = \sum^{N_k} \chi^2(k)}$
  - At this stage, we neglect the correlation between data at different k
- $\boxed{N = N_k + 6}$ fit parameters
- $\boxed{N_k N_t - N}$ degrees of freedom ($N_t = 7$)
- Errors on the fit parameters determined by finding the hypersurface with $\chi^2_{gl} - \chi^2_{gl(min)} = 1$

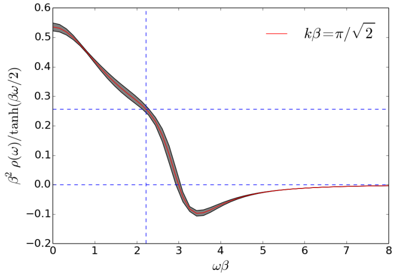
Results: correlator for the smallest χ^2_{gl} we found

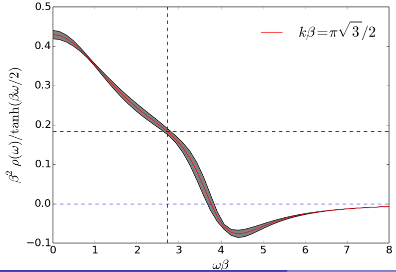


- 3 momenta fitted simultaneously: $k\beta = (\pi/2, \pi/\sqrt{2}, \sqrt{3}\pi/2)$
- $N_P = 9$ fit parameters
- $\mathrm{d.o.f.} = 12$
- $\chi^2_{gl}/\mathrm{d.o.f.} = 0.63$

Results: spectral function



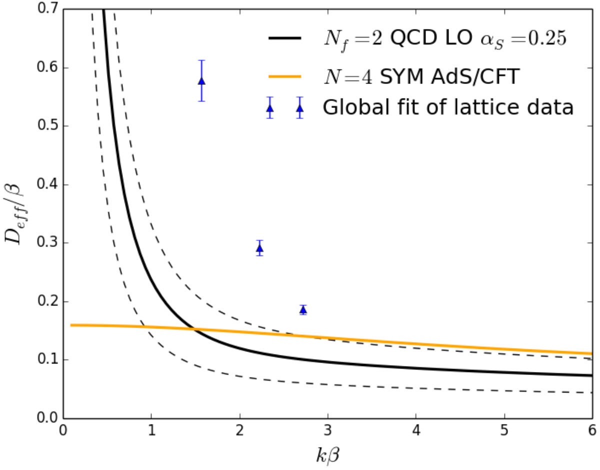




$$\chi^2_{gl}/\text{d.o.f.} = 0.63$$

Results: D_{eff}

$$\boxed{\chi^2_{gl}/\mathrm{d.o.f.} = 0.63}$$



Conclusions

- Global fits, including data at multiple momentum values, allow to significantly constrain the shape of the spectral function and the value of the photon rate
- However, we cannot exclude the existence of other local minima of χ^2_{gl}, characterized by very different values of these observables
- We plan on computing our observables around other local minima of χ^2_{gl}, and on extending our analysis to all the momentum values available
- A Euclidean correlator at zero virtuality ($\rightarrow$ imaginary spatial momentum) can be used to exclusively probe the photon rate, rather than the full (ω, k) dependence