Exercise sheet 8
Theoretical Physics 6a (QFT): SS 2019

3.6.2019

Exercise 1. (50 points) : Scalar 2 $\rightarrow$ 4 scattering

Considering the interaction Lagrangian for scalar fields

$$\mathcal{L}_1 = -\frac{\lambda}{4!}\phi^4, \tag{1}$$

and the Dyson Expansion of the S-Matrix:

$$S = \sum_{n=0}^{\infty} \frac{(-i)^n}{n!} \int d^4x_1 \cdots \int d^4x_n T\left\{\mathcal{H}_1(x_1)\cdots\mathcal{H}_1(x_n)\right\}. \tag{2}$$

Calculate the second order ($n = 2$) S-matrix element for a process of 2 initial bosons (of momenta p_1 and p_2) going to 4 final ones (of momenta p_3, p_4, p_5 and p_6) by using Wicks theorem and draw the diagrams which arise from it (at least 2 re-orderings of the external fields).

Exercise 2. (50 points) : Scalar QED

The Lagrangian of scalar QED is given by:

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + (D_\mu\phi)^*(D^\mu\phi) - m^2\phi^*\phi, \tag{3}$$

where $D_\mu = \partial_\mu + ieA_\mu$.

(a)(25 points) Derive the Feynman rules for scalar QED using the Lagrangian of Eq. (3) by identifying the interaction term and using Wicks theorem.

(b)(25 point) Use the Feynman rules to calculate the matrix elements for the two processes:

- $\gamma\gamma \to \phi^+\phi^-$
- $\gamma\phi^- \to \gamma\phi^-$.