Exercise sheet 11
Theoretical Physics 6a (QFT): SS 2019

24.06.2019

Exercise 1. (20 points) : Wick rotation

Consider the integral in D-dimensions:

$$I = \int \frac{d^D k}{(2\pi)^D} \frac{1}{(k^2 - \Delta + i\varepsilon)^2}.$$

In the lectures we performed this integral using the Wick rotation with $\Delta > 0$. Go through the same steps as in the lecture notes to show that the Wick rotation method still works for $\Delta < 0$. What is the difference between both cases?

Exercise 2. (40 points) : Pauli-Villars regularization

Regularize the 4 dimensional integral

$$I = \int \frac{d^4 k}{(2\pi)^4} \frac{1}{(k^2 - \Delta + i\varepsilon)^2}, \tag{1}$$

using the Pauli-Villars regularization as:

$$I \to \int \frac{d^4 k}{(2\pi)^4} \left\{ \frac{1}{(k^2 - \Delta + i\varepsilon)^2} - \frac{1}{(k^2 - \Lambda^2 + i\varepsilon)^2} \right\}, \tag{2}$$

with fixed mass scale $\Lambda^2 \gg \Delta$.

(a)(30 points)

Using Wick rotation, show that the 4 dimensional integral of Eq. (2) is given by:

$$I = -\frac{i}{(4\pi)^2} \ln \frac{\Delta}{\Lambda^2}.$$

(b)(10 points)
Compare this result with the corresponding one using dimensional regularization (expression from the lecture notes). Which identification on Λ has to be made to obtain the result of dimensional regularization ?

Exercise 3. (40 points) : Feynman parameterizations

Prove the following generalizations of the Feynman parametrization (by explicitly working out the integrals on the right hand sides):

(a)(20 points)

$$\frac{1}{A_1 A_2 A_3} = \Gamma(3) \int_0^1 dz_1 \int_0^{z_1} dz_2 \frac{1}{[A_1 + (A_2 - A_1) z_1 + (A_3 - A_2) z_2]^3}.$$

(b)(20 points)

$$\frac{1}{A^\alpha B^\beta} = \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} \int_0^1 dx \frac{x^{\alpha-1}(1-x)^{\beta-1}}{[B + (A - B)x]^{\alpha+\beta}}.$$