Exercise sheet 10
Theoretical Physics 6a (QFT): SS 2019

17.06.2019

Exercise 1. (50 points) : $\gamma\gamma \to e^+e^-$

(a)(15 points) Calculate the second order S-matrix element (S_{fi}) for $\gamma\gamma \to e^+e^-$ and draw the 2 contributing Feynman diagrams. Denote the incoming momenta as p_1 and p_2, and the outgoing momenta as p_3 and p_4.

(b)(15 point) Show that, after averaging over initial spins and summing over final ones, the unpolarized squared amplitude can be written as:

$$
\begin{aligned}
\overline{\sum_i}\sum_f |\mathcal{M}_0|^2 =& \frac{1}{4}\sum_i\sum_f (\mathcal{M}_{\mu\nu})^* \mathcal{M}^{\mu\nu} \\
&= \frac{1}{4}e^4 \left\{ \frac{\mathrm{Tr}\left[(\not{p}_3+m)\gamma^\mu(\not{p}_3-\not{p}_1+m)\gamma^\nu(\not{p}_4-m)\gamma_\nu(\not{p}_3-\not{p}_1+m)\gamma_\mu\right]}{(t-m^2)^2} \right. \\
&\quad + \frac{\mathrm{Tr}\left[(\not{p}_3+m)\gamma^\nu(\not{p}_1-\not{p}_4+m)\gamma^\mu(\not{p}_4-m)\gamma_\mu(\not{p}_1-\not{p}_4+m)\gamma_\nu\right]}{(u-m^2)^2} \\
&\quad \left. + \frac{2\,\mathrm{Tr}\left[(\not{p}_3+m)\gamma^\mu(\not{p}_3-\not{p}_1+m)\gamma^\nu(\not{p}_4-m)\gamma_\mu(\not{p}_1-\not{p}_4+m)\gamma_\nu\right]}{(t-m^2)(u-m^2)} \right\}.
\end{aligned}
$$

(c)(20 points) Show that the invariant differential cross section

$$
\frac{d\sigma}{dt} = \frac{1}{64\pi s}\frac{1}{|\vec{p}_{1\mathrm{cm}}|^2}\overline{\sum_i}\sum_f |\mathcal{M}_0|^2
$$

is given in terms of the Mandelstam variables

$$(p_1 + p_2)^2 = s$$
$$(p_1 - p_3)^2 = t$$
$$(p_2 - p_3)^2 = u$$

by

$$\frac{d\sigma}{dt} = \frac{e^4}{8\pi s^2}\left\{\frac{m^2 s - st - t^2 - 3m^4}{(t-m^2)^2} + \frac{m^2 s - su - u^2 - 3m^4}{(u-m^2)^2} + \frac{2m^2\left(s - 4m^2\right)}{(t-m^2)(u-m^2)}\right\}.$$

Exercise 2. (50 points) : Compton Scattering

(a)(10 points) For the elastic collision between a photon and an electron, $\gamma + e^- \to \gamma + e^-$, express the energy of the outgoing photon in lab system (ω') as function of the electron mass (m_e), the initial photon energy (ω) and the photon scattering angle (θ) (all in the lab system).

(b)(15 points) Calculate the second order S-matrix element (S_{fi}) for the Compton scattering off an electron and draw all the possible Feynman diagrams.

(c)(15 points) Show that the unpolarized differential cross section for the Compton scattering in the lab system can be written as

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{lab}} = \frac{\alpha^2}{2m_e^2}\left(\frac{\omega'}{\omega}\right)^2\left\{\frac{\omega}{\omega'} + \frac{\omega'}{\omega} - \sin^2\theta\right\}.$$

(d)(10 points) What is the numerical value of the differential cross section (in nanobarns) when the photon energy $E_{\text{lab}} = 1$ GeV and the lab scattering angle is $90°$.