

Effective Field Theory

Bottom-Up

Series 3

Assignment 1:

Let us look at QED for energies well below the electron mass. The effective theory then only consists of photons.

- (a) Write down the most general Lagrangian at mass dimension four, which respects all symmetries of QED. Note that the constructed Lagrangian should be invariant under the gauge transformation of the photon field A_μ

$$A_\mu \rightarrow A_\mu + \partial_\mu \chi. \quad (1)$$

- (b) Explain why there are no dimension six operators.
Hint: Use the Maxwell equations in the absence of external currents

$$\partial_\mu F^{\mu\nu} = 0, \quad (2)$$

$$\epsilon_{\mu\nu\sigma\eta} \partial^\nu F^{\sigma\eta} = 0. \quad (3)$$

Assignment 2:

The Lagrangian describing a system of a heavy and a light particle in the full theory is given by

$$\begin{aligned} \mathcal{L} = & \frac{1}{2} \partial_\mu \phi_L \partial^\mu \phi_L - \frac{m^2}{2} \phi_L^2 + \frac{1}{2} \partial_\mu \phi_H \partial^\mu \phi_H - \frac{M^2}{2} \phi_H^2 - \frac{\lambda_L}{4!} \phi_L^4 - \frac{\lambda_H}{4!} \phi_H^4 \\ & - \frac{\lambda_{HL}}{2!2!} \phi_L^2 \phi_H^2 - \frac{g}{2} \phi_H \phi_L^2. \end{aligned} \quad (4)$$

- (a) Show that the effective Lagrangian to order $\frac{1}{M^4}$ has the following form

$$\mathcal{L}_{\text{eff}} = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{\tilde{m}^2}{2} \phi^2 - \frac{\tilde{\lambda}}{4!} \phi^4 - \frac{C}{M^2} \phi \square^2 \phi - \frac{\tilde{C}}{M^2} \phi^2 \square \phi^2 - \frac{C'}{M^2} \phi^6 \quad (5)$$

- (b) Perform the following field redefinition

$$\phi \rightarrow \phi + \frac{\alpha}{M^2} \square \phi + \frac{\beta}{M^2} \phi^3 \quad (6)$$

and calculate the change in the effective lagrangian dropping terms of $\mathcal{O}(1/M^4)$.

- (c) Fix the coefficients α and β such that the two operators $\phi \square^2 \phi$ and $\phi^2 \square \phi^2$ are removed from \mathcal{L}_{eff} .
- (d) Draw all diagrams to one loop order that contribute to the two point function of the light particle in the full and effective theory.
- (d) Draw all diagrams to one loop order that contribute to the four point function of the light particle in the full and effective theory.