Effective Field Theory

Top-down approach **Series 2**

Assignment 1:

Derive the integral representation of the renormalization group equation

$$g(\mu) = g(\mu_1) \exp \int_{\ln \mu_1}^{\ln \mu} d \ln \mu' \gamma(\mu') \tag{1}$$

starting with

$$\frac{\partial}{\partial \ln \mu} g(\mu) = \gamma(\mu) g(\mu). \tag{2}$$

Suppose γ is known in perturbation theory and reads

$$\gamma(\alpha) = \gamma_0 \frac{\alpha}{4\pi} + \mathcal{O}(\alpha^2), \quad \frac{\partial \alpha}{\partial \ln \mu} = \beta(\alpha) = -2\alpha \Big[\beta_0 \frac{\alpha}{4\pi} + \mathcal{O}(\alpha^2) \Big] \tag{3}$$

Show that

$$g(\mu) \simeq g(\mu_1) \Big(\frac{\alpha(\mu)}{\alpha(\mu_1)} \Big)^{-\frac{\gamma_0}{2\beta_0}}. \tag{4}$$

Assignment 2:

Calculate the one-loop self energy correction from integrating out the heavy modes in the scalar theory

$$\mathcal{L} = \frac{1}{2} \Big[\partial_\mu \phi \partial^\mu \phi - m^2 \phi^2 \Big] - \frac{\lambda}{4!} \phi^4 \tag{5}$$

What is the change in mass in going from a theory integrated out at Λ_1 and at $\Lambda_2 > \Lambda_1$? The only diagram contributing is

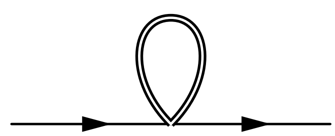

Figure 1: Scalar theory self energy contribution, where the double line denotes the heavy degree of freedom

Use

$$\int dk \frac{k^3}{k^2 - m^2} = \frac{1}{2} \left(m^2 \log \left(1 - \frac{k^2}{m^2} \right) + k^2 \right) \tag{6}$$

Redo the calculation in $\overline{\text{MS}}$. Note that

$$\frac{i\mu^{4-D}}{(2\pi)^D} \int d^D k \frac{1}{k^2 - m^2} = -\frac{1}{16\pi^2} m^2 \Big[\Delta + \ln \frac{\mu^2}{m^2} + 1 \Big] \tag{7}$$

$$\Delta = \frac{2}{4-D} - \gamma_{\text{E}} + \ln 4\pi \tag{8}$$

What is the relation between the β function and the Wilsonian RGE solution?