Chapter 1

Introduction

What is remarkable about physics is that you do NOT need the theory of everything to do useful stuff.
In your physics career (so far) you have learned more and more complex theories, with increasing difficulty. In this course we will learn how to make life easier again.
This is the essence of an effective field theory:
Cover all phenomena you are interested in using the **simplest** possible model.
First and foremost EFT is a Quantum Field Theory (QFT) which:

- concentrates on the relevant degrees of freedom (easier theory)
- allows for systematic improvements (more than just the limit of sth. else)

In practice:
EFT is a low energy (below some scale Λ) approximation of a more fundamental theory

- Λ separates relevant from irrelevant degrees of freedom
- Physics above Λ is encoded in EFT via
    1. symmetries (interactions)
    2. Low Energy Constants (organized as parameters of $\left(\frac{E}{\Lambda}\right)^n$)

Two possible scenarios

1. we know high energy theory but things are hard to calculate
   top-down approach (Wilsonian EFT)

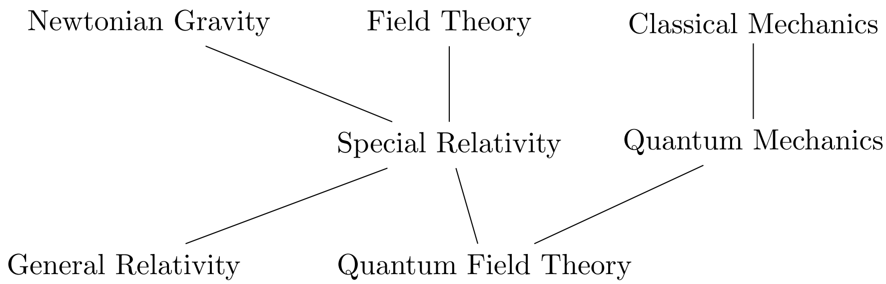

Figure 1.1: Going down the landscape of theories life gets harder.

- Integrating out heavy degrees of freedom
- Matching of LEC

2. We do not know the high energy theory, or too hard to calculate bottom-up approach (Continuum EFT)

   - construct most general theory consistent with given theory
   - infinite number of terms, need a guiding principle, i.e. power counting
   - fix LECs from experiment

Modern point of view

Every QFT is an EFT, i.e. restricted range of applicability. In this sense every QFT can be seen as the low energy approximation of a (possibly unknown) more fundamental theory. This includes the Standard Model, which for historical reasons was created to be a renormalizable theory, that is in the traditional sense.

Renormalizability in the traditional sense:

At any finite order in perturbation theory divergences from loop integrals can be absorbed in a finite numbers of parameters.
EFTs are not renormalizable in this sense!

Renormalizability in the EFT sense:

At any finite order in the expansion parameter $\frac{E}{\Lambda}$ only a finite number of parameters are need to render amplitudes finite.
For some examples of EFTs see Tab. 1.1.

This is not the only distinction, one can also follow the transition from the high energy to the low energy domain, which can happen via

Low Energy	High Energy	Scale
Standard Model	?	?
Fermitheory	Standard Model	$M_W \approx 80\text{GeV}$
Chiral Perturbation Theory	QCD	$4\pi F_\pi, m_\rho, m_p \approx 1\text{GeV}$

Table 1.1: Examples for effective field theories.

1. Complete decoupling:
   High energy theory contains heavy particle never produced if $E \ll M$
   Example (in more detail later):
   Weak interactions, e.g. β-decays

   $$n \to p e^- \overline{\nu}_e \tag{1.1}$$

   Expand the W-propagator

   $$\frac{1}{q^2 - M^2} \to -\frac{1}{M^2} \tag{1.2}$$

   This amounts to replacing the diagram with a four fermion vertex. The procedure, i.e. expanding non-local UV-physics in local operators of increasing dimensionality, is called integrating out the heavy modes. In the following we will look at an example at the classical level. Let us look at the Lagrangian density of scalar particles

   $$\begin{aligned} \mathcal{L} &= \mathcal{L}_0 + \mathcal{L}_{\text{int}} && (1.3)\\ \mathcal{L}_0 &= \frac{1}{2}(\partial_\mu \Phi \partial^\mu \Phi - M^2 \Phi^2) + \frac{1}{2}(\partial_\mu \varphi \partial^\mu \varphi - m^2 \varphi^2) && (1.4)\\ \mathcal{L}_{\text{int}} &= -\frac{\lambda}{2} \Phi \varphi^2 && (1.5) \end{aligned}$$

   leading to the equations of motion for Φ and φ

   $$\begin{aligned} \Box \Phi + M^2 \Phi + \frac{\lambda}{2} \varphi^2 &= 0 && (1.6)\\ \Box \varphi + m^2 \varphi + \lambda \varphi \Phi &= 0 && (1.7) \end{aligned}$$

   Now from Eq. (1.6) we get

   $$\Phi = -\frac{\lambda}{2M^2} \frac{1}{1 + \frac{\Box}{M^2}} \varphi^2 \tag{1.8}$$

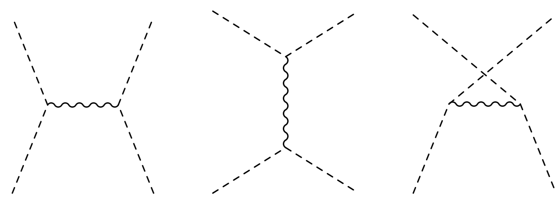

Figure 1.2: Diagrams contributing to $\varphi\varphi$ scattering in the full theory.

insert into Eq. (1.7)

$$0 = \Box\varphi + m^2\varphi + \lambda\varphi\Big(-\frac{\lambda}{2M^2}\frac{1}{1+\frac{\Box}{M^2}}\varphi^2\Big) \tag{1.9}$$

Expand to leading order in $1/M^2$

$$0 = \Box\varphi + m^2\varphi - \frac{\lambda^2}{2M^2}\varphi^3 \tag{1.10}$$

Now the question is which **effective** Lagrangian would generate the EOM of Eq. (1.10)?

$$\mathcal{L}_{\text{eff}} = \frac{1}{2}\Big(\partial_\mu\varphi\partial^\mu\varphi - m^2\varphi^2\Big) + \frac{\lambda^2}{8M^2}\varphi^4 \tag{1.11}$$

We observe the following:

- Φ is gone completely
- New φ^4 interaction
- New interaction suppressed by $1/M^2$

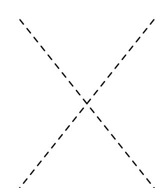

Figure 1.3: Diagrams contributing to $\varphi\varphi$ scattering in the effective theory.

Let's look at $\varphi\varphi$ scattering, i.e. $\mathcal{M}(\varphi\varphi \to \varphi\varphi)$ in the full theory this reads

$$\mathcal{M} = (-i\lambda^2)\Big(\frac{i}{s-M^2} + \frac{i}{t-M^2} + \frac{i}{u-M^2}\Big) \tag{1.12}$$

where s, t, u are the Mandelstam variables for a $2 \to 2$ scattering process. Expanding the matrix element in powers of $1/M^2$ we get for the low energy region where s,t, u are all small compared to M

$$\mathcal{M} \approx (-i\lambda^2)\Big(-\frac{3i}{M^2} + \mathcal{O}\Big(\frac{\{s,t,u\}}{M^4}\Big)\Big) \tag{1.13}$$

$$= \frac{3i\lambda^2}{M^2} \tag{1.14}$$

Now the same calculation in EFT using the effective Lagrangian only needs one diagram, where the Feynman rule introduces a 4! for the possible permutations of external legs.

$$\mathcal{M} = \frac{i\lambda^2}{8M^2}4! = \frac{3i\lambda^2}{M^2} \tag{1.15}$$

2. Partial Decoupling
   Heavy fields are still present in theory, e.g. high momentum modes have been removed.
   Example: HQET

3. Spontaneous Symmetry Breaking
   In the transition from high to low energy SSB happens, giving rise to massless excitations (Goldstone Bosons). EFTs are formulated in terms of GSB, i.e. particle content changes at low energy.
   Example: χPT

A word on units:
We will use natural units $\hbar = c = 1$ throughout.
This means that everything will have dimension of energy.

$$c = \lambda\nu \qquad [\hbar\nu] = [\text{Energy}] \qquad [\text{mc}^2] = [\text{Energy}] \qquad [\lambda] = [\text{Energy}^{-1}]$$