Exercise sheet 7
Theoretical Physics 6a (QFT): SS 2019

27.05.2019

Exercise 1. (40 points) : Scalar Quantum Electrodynamics

Consider the Lagrangian for the charged Klein-Gordon Field

$$\mathcal{L} = (\partial_\mu \phi)^\dagger (\partial^\mu \phi) - m^2 \phi^\dagger \phi \tag{1}$$

- Check that this Lagrangian is not invariant under U(1) gauge transformation of the form $\phi(x) \to \phi'(x) = e^{i\alpha(x)}\phi(x)$;
- Replace the derivatives by the convariant one $D_\mu = \partial_\mu + ieA_\mu$ and check whether now the Lagrangian is invariant under the local symmetry;
- Write down the interaction Lagrangian and identify the terms. Sketch each one as a different diagram;

Exercise 2. (40 points) : Gauge transformation

Consider a state $|\Psi_T\rangle$ which only contains transverse photons. Futhermore, construct a state $|\Psi_T'\rangle$ as:

$$|\Psi_T'\rangle = \left\{1 + \alpha \left[a^\dagger(\vec{k}, 3) - a^\dagger(\vec{k}, 0)\right]\right\} |\Psi_T\rangle,$$

with α a constant. Show that replacing $|\Psi_T\rangle$ by $|\Psi'_T\rangle$ corresponds to a gauge transformation:

$$\langle\Psi'_T|A^\mu(x)|\Psi'_T\rangle = \langle\Psi_T|A^\mu(x) + \partial^\mu\Lambda|\Psi_T\rangle \,,$$

where Λ is given by:

$$\Lambda(x) = \mathrm{Re}\left(i\alpha\frac{\sqrt{2}}{\omega_k^{3/2}}e^{-ik\cdot x}\right) .$$

Exercise 3. (20 points) : Positronium

Positronium is a bound state of an e^- and an e^+. Their spins can combine into either total spin $S = 0$ or 1.

(a)(5 points) Show that the corresponding spin wavefunctions are either odd ($S = 0$) or even ($S = 1$) under exchange of the spins.

(b)(5 points) Write down all allowed total angular momentum values for a positronium state which has orbital angular momentum $L = 0$ or $L = 1$.

(c)(5 points) Show that for a state of total spin S and orbital angular momentum L, the parity P of the state (eigenvalue of the P operation) is given by $P = (-1)^{L+1}$.

(d)(5 points) Show that for a state of total spin S and orbital angular momentum L, the C-parity C of the state (eigenvalue of the charge conjugation C operation) is given by $C = (-1)^{L+S}$.