Effective Field Theory

Lagrangian Formalism **Series 1**

Assignment 1:

Derive the equations of motion for the following Lagrangian densities

(a)* free Dirac field

$$\mathcal{L} = \overline{\Psi}\Big(i\gamma_\mu\partial^\mu - m\Big)\Psi \tag{1}$$

for the field Ψ. Note that

$$\overline{\Psi} = \Psi^\dagger\gamma_0 \tag{2}$$

$$\gamma_\mu^\dagger = \gamma_0\gamma_\mu\gamma_0 \tag{3}$$

(b) electromagnetic interaction

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + j^\mu A_\mu \tag{4}$$

with

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \tag{5}$$

for the field A_μ.

Assignment 2 *:

Calculate the conserved current for a Lagrangian density with quartic coupling, i.e.

$$\mathcal{L} = \frac{1}{2}\Big[\partial_\mu\Phi\partial^\mu\Phi + \partial_\mu\varphi\partial^\mu\varphi - m^2(\Phi^2+\varphi^2)\Big] - \frac{\lambda}{4}\Big(\Phi^2+\varphi^2\Big)^2 \tag{6}$$

under active rotation of the fields by an angle $\varepsilon(x)$

$$\Phi(x)' = \Phi(x) - \varepsilon(x)\varphi(x) \qquad \varphi(x)' = \varphi(x) + \varepsilon(x)\Phi(x). \tag{7}$$

The current and its divergence are given by

$$J^\mu = \frac{\partial\delta\mathcal{L}}{\partial\partial_\mu\varepsilon}, \quad \partial_\mu J^\mu = \frac{\partial\delta\mathcal{L}}{\partial\varepsilon}. \tag{8}$$

Can you explain why this is an infinitesimal rotation?
Do you know the corresponding symmetry group?

Assignment 3:

Using the Hamiltonian

$$H = \frac{1}{2}\int\frac{dk}{(2\pi)^3 2k_0}k_0\Big[a^\dagger(\mathbf{k})a(\mathbf{k}) + a(\mathbf{k})a^\dagger(\mathbf{k})\Big] \tag{9}$$

(a) Calculate

$$Ha(\mathbf{k})|E\rangle = (a(\mathbf{k})H + [H, a(\mathbf{k})])|E\rangle \tag{10}$$

using the commutation relations

$$[a(\mathbf{k}'), a(\mathbf{k})] = [a^\dagger(\mathbf{k}'), a^\dagger(\mathbf{k})] = 0 \tag{11}$$

$$[a^\dagger(\mathbf{k}'), a(\mathbf{k})] = -(2\pi)^3 2k_0 \delta^3(\mathbf{k} - \mathbf{k}') \tag{12}$$

where $|E\rangle$ are energy eigenstates of the Hamiltonian with eigenvalues e.

(b) Using the relation in Eq. (12) rewrite the Hamiltonian of Eq. (10) such that all creation operators $a^\dagger$ are to the left. Compute the integral using a cutoff Λ for the momentum $|\mathbf{k}|$.

Assignment 4 *:

(a) What are the mass dimensions of the fields Ψ, A and φ of Assignment 1/2? Using these mass dimensions determine if the dimension of the following operators in D dimensions

$$\mathcal{O}_1 = \varphi\overline{\Psi}\Psi \quad \mathcal{O}_2 = \overline{\Psi}\gamma_\mu A^\mu \Psi \quad \mathcal{O}_3 = \overline{\Psi}\Psi\overline{\Psi}\Psi \tag{13}$$

(b) Let the four fermion operator have a coupling of size $1.16 \times 10^{-5}\mathrm{GeV}^{-2}$. At what scale would that interaction be of natural size.

Assignment 5:

This has nothing to do with Lagrangian formalism (however with dimensional regularization).
Calculate the D-dimensional surface of the unit sphere. Use the following

$$\Big[\int_{-\infty}^{\infty} dx e^{-x^2}\Big]^D = \int_{-\infty}^{\infty} d^D x e^{-x^2} = \Omega_D \int dr r^{D-1} e^{-r^2} \tag{14}$$

Note that looking up the definition of the Γ function might prove useful.
You should hand in (at least) the assignments with an asterisk.

Please hand in the exercise 29.04. in the lecture.