Introduction to Lattice Gauge Theory

PD Dr. Georg von Hippel

JOHANNES GUTENBERG UNIVERSITÄT MAINZ

Sommersemester 2019

Syllabus: Organisation

JOGUStINe: Registration

The registration deadline is **Thursday, 18th April 2019 at 21:00**.

Only participants registered in JOGUStINe by then can receive credit for this course.

Syllabus: Organisation

Examination

The examination mode (*Leistungsnachweis*) for this course is an **oral exam** of 30 minutes duration.

To qualify for the exam, participants have to participate actively in the examples classes (*Übungen*) and reach at least 50% of the achievable marks on the homework sheets.

Syllabus: Organisation

Examples Classes

Examples classes will take place on **Thursdays, 10:00–12:00**, in the **Galilei-Raum**.

The classes are an integral part of the course and will develop new material. Attendance is essential even if you do not intend to take the course for credit.

Syllabus: Website

https://wwwth.kph.uni-mainz.de/ss2019-gittereichtheorie/

- Notifications regarding changes to the schedule
- Download area:
  - these slides
  - reading assignments
  - other course materials

Syllabus: Literature

G. Parisi, *Statistical Field Theory* (Frontiers in Physics **66**), Addison-Wesley, Redwood City 1988.

J.B. Kogut, *An Introduction to Lattice Gauge Theory and Spin Systems*, Rev. Mod. Phys. **51** (1979) 659.

C. Gattringer and C.B. Lang, *Quantum Chromodynamics on the Lattice* (Lect. Notes Phys. **788**), Springer, Berlin Heidelberg 2010.

J. Smit, *Introduction to Quantum Fields on a Lattice: a robust mate* (Cambridge Lect. Notes Phys. **15**), Cambridge University Press 2002.

From the "Particle Zoo" to Quarks

Gell-Mann showed in 1964 that the mass patterns of **hadrons** can be explained by uniting isospin and strangeness into a larger $\mathrm{SU}(3)$ flavour symmetry based on three **quarks** u, d, and s as basic building blocks.

MURRAY GELL-MANN
(1929–, Nobel Prize 1969)

A simpler and more elegant scheme can be constructed if we allow non-integral values for the charges. We can dispense entirely with the basic baryon b if we assign to the triplet t the following properties: spin $\frac{1}{2}$, $z = -\frac{1}{3}$, and baryon number $\frac{1}{3}$. We then refer to the members $u^{\frac{2}{3}}$, $d^{-\frac{1}{3}}$, and $s^{-\frac{1}{3}}$ of the triplet as "quarks" 6) q and the members of the anti-triplet as anti-quarks $\bar{q}$. Baryons can now be constructed from quarks by using the combinations $(q\,q\,q)$, $(q\,q\,q\,q\,\bar{q})$, etc., while mesons are made out of $(q\,\bar{q})$, $(q\,q\,\bar{q}\,\bar{q})$, etc. It is assuming that the lowest baryon configuration $(q\,q\,q)$ gives just the representations **1**, **8**, and **10** that have been observed, while the lowest meson configuration $(q\,\bar{q})$ similarly gives just **1** and **8**.

6) James Joyce, Finnegan's Wake (Viking Press, New York, 1939) p.383.

From the "Particle Zoo" to Quarks

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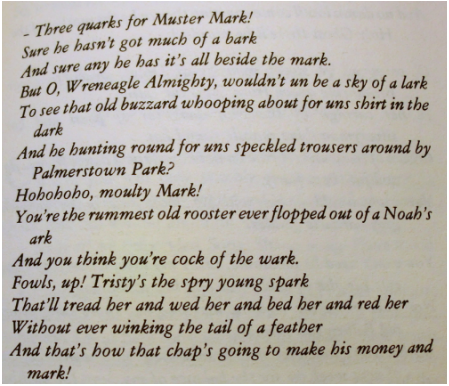
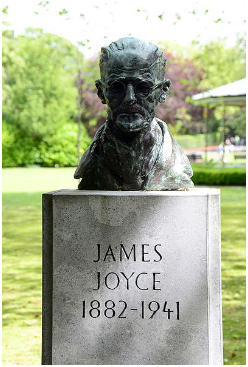


– Three quarks for Muster Mark!
Sure he hasn't got much of a bark
And sure any he has it's all beside the mark.
But O, Wreneagle Almighty, wouldn't un be a sky of a lark
To see that old buzzard whooping about for uns shirt in the dark
And he hunting round for uns speckled trousers around by Palmerstown Park?
Hohohoho, moulty Mark!
You're the rummest old rooster ever flopped out of a Noah's ark
And you think you're cock of the wark.
Fowls, up! Tristy's the spry young spark
That'll tread her and wed her and bed her and red her
Without ever winking the tail of a feather
And that's how that chap's going to make his money and mark!



From the “Particle Zoo” to Quarks

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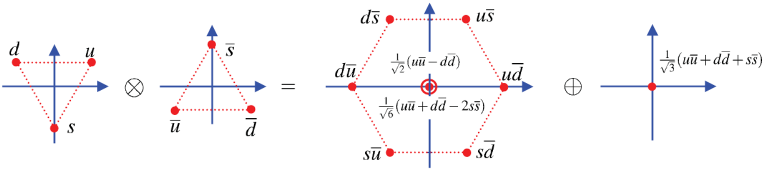
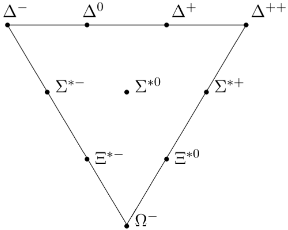


The rules of this “eightfold way” ($=$ representation theory of $\mathrm{SU}(3)$) can be seen as a generalization of angular momentum addition.

From Quarks to QCD



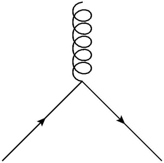
The existence of the Ω^- was a triumph of the resulting quark model, but it also indicated a problem with the model: the total symmetry of an S-wave spin-$\frac{3}{2}$ sss state contradicts Fermi statistics for the quarks, necessitating the introduction of a new "**colour**" quantum number under which the state could be antisymmetric.

Colour is not observed in experiments, so physical states have to be colour-neutral (colour confinement).

From Quarks to QCD

By coupling a new interaction to colour, strong force binding quarks into hadron might be explained. To do this consistently, must introduce new $\mathrm{SU}(3)$ **gauge symmetry** acting on colour *locally*, $\psi(x) \mapsto \Omega(x)\psi(x)$.

To do this consistently requires introduction of new field $A_\mu(x)$ entering covariant derivative $D_\mu = \partial_\mu + gA_\mu$ that replaces ∂_μ. To ensure covariance $D_\mu \mapsto \Omega(x)D_\mu$, A_μ must transform as $A_\mu(x) \mapsto \Omega(x)A_\mu(x)\Omega^{-1}(x) - \frac{1}{g}\left(\partial_\mu\Omega(x)\right)\Omega^{-1}(x)$.



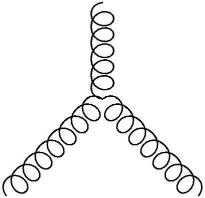
Quanta of A_μ are **gluons**, resulting QFT is **QCD** (Quantum Chromodynamics, <gr. χρῶμα "colour"). The gluons couple to quarks like photons do to electrons in QED, with the coupling g taking the place of the charge e.

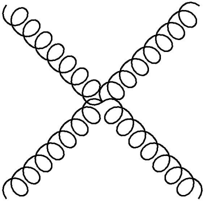
From Quarks to QCD

Big difference between QCD and QED: Gluons also couple to other gluons, since $\mathrm{SU}(3)$ is **non-abelian**:

$$F_{\mu\nu} = \frac{1}{g}[D_\mu, D_\nu] = \partial_\mu A_\nu - \partial_\nu A_\mu + g \underbrace{[A_\mu, A_\nu]}_{\neq 0}$$

and the kinetic term $\frac{1}{2}\,\mathrm{tr}\,(F_{\mu\nu}F^{\mu\nu})$ contains three-gluon and four-gluon vertices, with the form of the gluon self-interactions fully dictated by gauge symmetry.

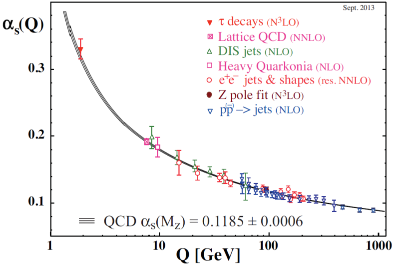




Asymptotic Freedom, Confinement, and the Lattice

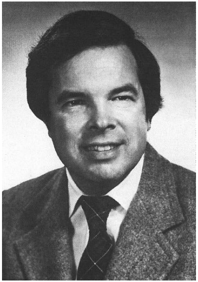
The self-interaction of the gluon field leads to a distinctive feature of QCD: instead of growing with energy as in QED, the strength of the interactions decreases with increasing energy (**asymptotic freedom**). At high energies, QCD is perturbative!

D. Politzer, D. Gross, F. Wilczek
(Nobel Prize 2004)



Asymptotic Freedom, Confinement, and the Lattice

The converse of asymptotic freedom is that the coupling gets strong at low energies. Perturbation theory is useless in understanding how quarks form hadrons, and why quarks and gluons are not observed in nature (**confinement**). A non-perturbative method to treat QCD is needed.



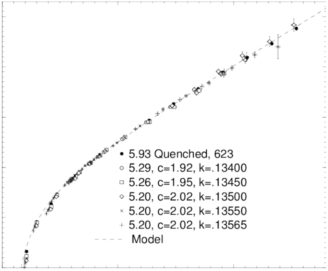
KENNETH WILSON
(1936–2013, Nobel Prize 1982)

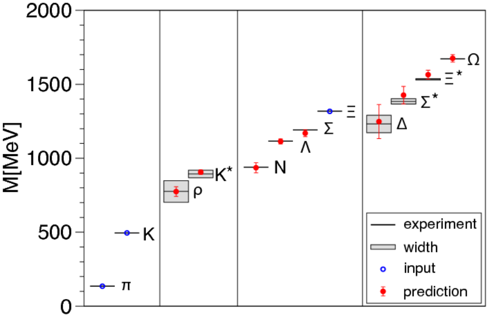
In 1974, Ken Wilson constructed the lattice formulation of QCD and showed that it permitted a strong-coupling expansion, in which quarks were confined.

Moreover, **Lattice QCD** could be simulated on a computer.

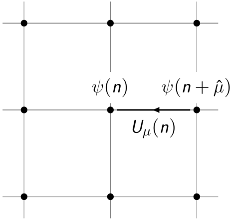
Asymptotic Freedom, Confinement, and the Lattice

Nowadays, Lattice QCD is the leading (and arguably only) method for treating the non-perturbative regime of QCD in order to understand quark confinement and predict the spectrum and other properties of hadrons from first principles.





A Bird's-Eye View of Lattice Field Theory



Matter fields (quarks, ...) live on the sites of a hypercubic lattice, gauge fields live on the links between sites. Derivatives are replaced by finite differences, the path integral becomes an ordinary (very high-dimensional) integral.

In essence, the lattice field theory can be seen as a 4D model in statistical mechanics. For analytic treatment, can expand around both weak and strong coupling limits. Non-perturbative results can be obtained by evaluating the path integral using Monte Carlo methods.

Syllabus: Course Outline

- Discretization of classical field theories
  - Finite differences
  - Fourier transform on the lattice
- The Feynman path integral and the transfer operator
- Discrete models
- Gauge symmetry
- Lattice QCD

Syllabus: Course Outline

- Discretization of classical field theories
- The Feynman path integral and the transfer operator
  - Path integral in quantum mechanics
  - Connection to statistical mechanics
  - Transfer operator and Hamiltonian
  - Solution of free field theories
- Discrete models
- Gauge symmetry
- Lattice QCD

Syllabus: Course Outline

- Discretization of classical field theories
- The Feynman path integral and the transfer operator
- Discrete models
  - Ising model
    - Exact solutions
    - High- and low-temperature expansions
    - Real-space renormalization group
    - Markov-Chain Monte Carlo
  - Ising gauge model
    - Elitzur's theorem
    - Area law and perimeter law
- Gauge symmetry
- Lattice QCD

Syllabus: Course Outline

- Discretization of classical field theories
- The Feynman path integral and the transfer operator
- Discrete models
- Gauge symmetry
  - Continuum gauge theories
  - Lattice formulation
  - Haar measure
  - Transfer operator and static potential
  - Area law and confinement at strong coupling
  - Perturbative expansion at weak coupling
- Lattice QCD

Syllabus: Course Outline

- Discretization of classical field theories
- The Feynman path integral and the transfer operator
- Discrete models
- Gauge symmetry
- Lattice QCD
  - Fermions on the lattice
    - Grassmann analysis
    - Nielsen-Ninomiya theorem
    - Wilson fermions
  - Monte Carlo simulations
    - Pseudofermions
    - Molecular dynamics and HMC algorithm
    - Autocorrelations
    - Measurement of hadronic masses
  - Advanced topics
    - as time permits . . .