

Theoretical Physics 6a (QFT): SS 2019
Exercise sheet 1

15.04.2019

Exercise 1 (100 points): Scalar theory with $SO(2)$ invariance

Consider the following Lagrangian density of two real scalar fields $\phi_1(x)$, $\phi_2(x)$:

$$\mathcal{L} = \frac{1}{2} \left[(\partial\phi_1)^2 + (\partial\phi_2)^2 \right] - \frac{m^2}{2} (\phi_1^2 + \phi_2^2) - \frac{\lambda}{4!} (\phi_1^2 + \phi_2^2)^2$$

(a)(20 points) Identify the corresponding equations of motion.

(b)(20 points) Show that the above Lagrangian is invariant under the transformations

$$\begin{aligned}\phi_1 &\rightarrow \phi'_1 = \phi_1 \cos \theta - \phi_2 \sin \theta, \\ \phi_2 &\rightarrow \phi'_2 = \phi_1 \sin \theta + \phi_2 \cos \theta.\end{aligned}$$

(c)(20 points) Calculate the Noether current j_μ and show explicitly that its divergence vanishes for fields ϕ_i which satisfy equations of motion.

(d)(20 points) Show explicitly that the Noether charge Q is a conserved quantity, assuming the surface integral $\int dS \vec{n} \cdot \vec{j}$ vanishes.

(e)(20 points) Construct the Hamiltonian density \mathcal{H} .