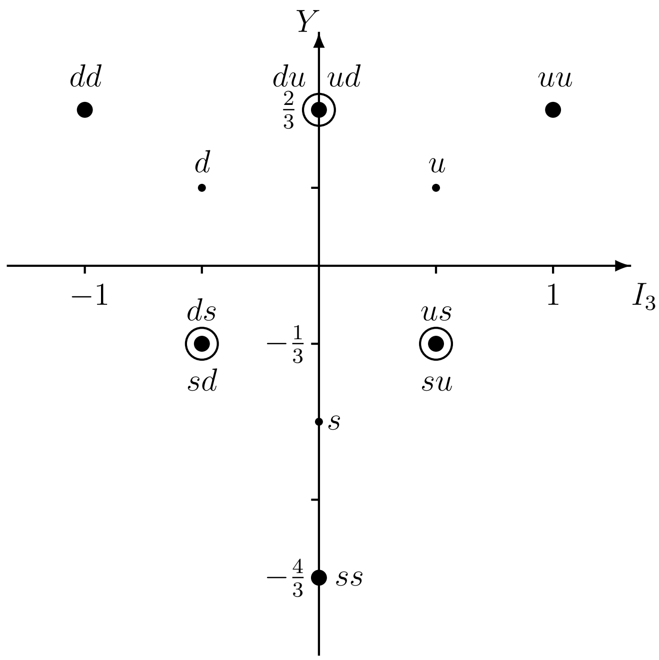
Handout 15 (read by Feb. 12)

5.4.8 Graphical construction.

Similarly as in the case of SU(2), we may construct the multiplets of the product representations of SU(3) as superpositions of the multiplets of the fundamental representation **3** and the corresponding conjugate representation $\bar{\mathbf{3}}$. In this case, because of the additivity of the I_3 and Y eigenvalues, we overlay the individual points of the first multiplet with the center $(I_3, Y) = (0, 0)$ of the second multiplet.

1. Let us start with the direct product of two fundamental triplets: $\mathbf{3} \otimes \mathbf{3} = \mathbf{6} \oplus \bar{\mathbf{3}}$.



In the figure, the first multiplet (u, d, s) is denoted by the small filled circles. The states of the product are denoted by the large filled circles and the rings. In analogy to the construction of the isospin states corresponding to $I = 1$ and $I = 0$, the corresponding states of the sextet **6** are symmetric and the states of the antitriplet $\bar{\mathbf{3}}$ are antisymmetric under the exchange of two quarks, respectively. To be specific, the states of the sextet **6** are given by:

$I = 1, Y = \frac{2}{3}$:

	uu	$\frac{1}{\sqrt{2}}(ud + du)$	dd
I_3	1	0	-1

$I = \frac{1}{2}, Y = -\frac{1}{3}$:

	$\frac{1}{\sqrt{2}}(us + su)$	$\frac{1}{\sqrt{2}}(ds + sd)$
I_3	$\frac{1}{2}$	$-\frac{1}{2}$

$I = 0, Y = -\frac{4}{3}$:

	ss
I_3	0

On the other hand, the states of the antitriplet $\bar{\mathbf{3}}$ are given by:

$I = \dfrac{1}{2}, Y = -\dfrac{1}{3}$:

	$\frac{1}{\sqrt{2}}(us - su)$	$\frac{1}{\sqrt{2}}(ds - sd)$
I_3	$\frac{1}{2}$	$-\frac{1}{2}$

$I = 0, Y = \dfrac{2}{3}$:

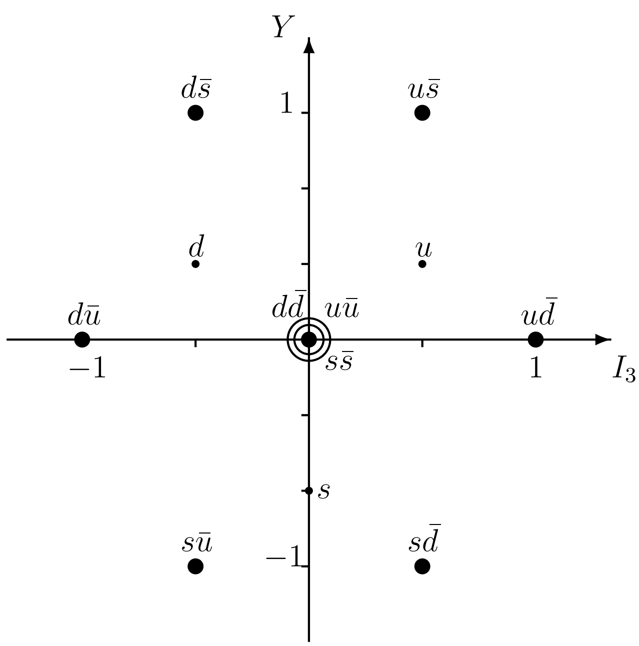
	$\frac{1}{\sqrt{2}}(ud - du)$
I_3	0

2. Remark: the weight diagrams of irreducible representations satisfy the following property. The points at the edges are singly occupied. In the next inner layer, the points are doubly occupied, etc.

   (a) For $\alpha = \beta$, the procedure ends in the origin. The origin is $(\alpha + 1)$-fold occupied.

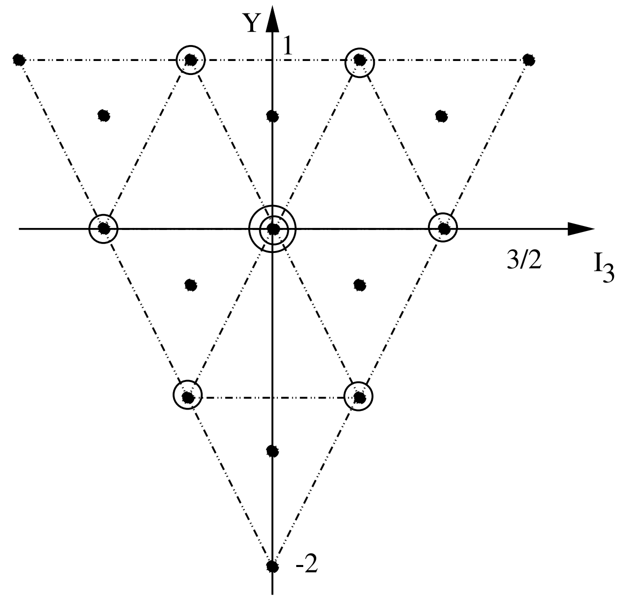
   (b) For $\alpha \neq \beta$, the procedure ends, once a triangular form has been reached. For $\alpha > \beta$, the points of the triangle (border and interior) are each $(\beta + 1)$-fold occupied. For $\beta > \alpha$, the points of the triangle (border and interior) are each $(\alpha + 1)$-fold occupied.

3. $\mathbf{3} \otimes \bar{\mathbf{3}} = \mathbf{8} \oplus \mathbf{1}$



Consider the octet $\mathbf{8}$. In this case we have $\alpha = 1$ and $\beta = 1$, i.e., the six points of the border are singly occupied, the center (next inner layer) is doubly occupied. The third state at the center is the singlet $\mathbf{1}$.

4. $\mathbf{6} \otimes \mathbf{3} = \mathbf{10} \oplus \mathbf{8}$



Consider the decuplet **10**. In this case we have $\alpha = 3$ and $\beta = 0$, i.e., the decuplet has a triangular shape and the points are singly occupied.

5. $(\mathbf{3} \otimes \mathbf{3}) \otimes \mathbf{3} = (\mathbf{6} \oplus \bar{\mathbf{3}}) \otimes \mathbf{3} = \mathbf{10} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{1}$