SU(N) and quarks 1

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A simpler and more elegant scheme can be constructed if we allow non-integral values for the charges. We can dispense entirely with the basic baryon b if we assign to the triplet t the following properties: spin $\frac{1}{2}$, $z = -\frac{1}{3}$, and baryon number $\frac{1}{3}$. We then refer to the members $u^{\frac{2}{3}}$, $d^{-\frac{1}{3}}$, and $s^{-\frac{1}{3}}$ of the triplet as "quarks" 6) q and the members of the anti-triplet as anti-quarks $\bar{q}$. Baryons can now be constructed from quarks by using the combinations $(q\,q\,q)$, $(q\,q\,q\,q\,\bar{q})$, etc., while mesons are made out of $(q\,\bar{q})$, $(q\,q\,\bar{q}\,\bar{q})$, etc. It is assuming that the lowest baryon configuration $(q\,q\,q)$ gives just the representations **1**, **8**, and **10** that have been observed, while the lowest meson configuration $(q\,\bar{q})$ similarly gives just **1** and **8**.

6) James Joyce, Finnegan's Wake: „three quarks for Muster Mark“

Physical motivation

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   QCD is a non-Abelian gauge theory with gauge group $G = \mathrm{SU}(3)_c$ (c for *color*)

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4. Matter fields of QCD (quarks) are fermions with spin 1/2, which show up in six different *flavors*

Light quarks

flavor	u	d	s
mass [MeV]	$2.2^{+0.6}_{-0.4}$	$4.7^{+0.5}_{-0.4}$	96^{+8}_{-4}
charge [$e > 0$]	$\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$
I_3	$+\frac{1}{2}$	$-\frac{1}{2}$	0
			strangeness: -1

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Heavy quarks

flavor	c	b	t
mass [GeV]	1.28 ± 0.03	$4.18^{+0.04}_{-0.03}$	173.1 ± 0.6
charge [$e > 0$]	$\frac{2}{3}$	$-\frac{1}{3}$	$\frac{2}{3}$
I_3	0	0	0
	charm: $+1$	bottom: -1	top: $+1$

See `http://pdg.lbl.gov`

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   - General N_c:
   $$\frac{1}{\sqrt{N_c!}} \epsilon_{i_1 \ldots i_{N_c}} \chi^{i_1} \otimes \ldots \otimes \chi^{i_{N_c}}$$

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11. Mesons: $q\bar{q}$ (quark-antiquark) states; color neutral via $\frac{1}{\sqrt{3}}(r\bar{r} + g\bar{g} + b\bar{b})$