SU($N$) and quarks 1

**Aim: Classification of composite states**

1. **Atoms**: Atomic nucleus and electron shell

- Nuclei: Protons and neutrons (Collective term: Nucleons $\subset$ Baryons $\subset$ Hadrons)

- Nucleons: Quarks (M. Gell-Mann, Phys. Lett. 8, 214 (1964))

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James Joyce, *Finnegan’s Wake*: “three quarks for Muster Mark”
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Aim: Classification of composite states

1. Atoms: Atomic nucleus and electron shell
2. Nuclei: Protons and neutrons (Collective term: Nucleons ⊂ baryons ⊂ hadrons)

A simpler and more elegant scheme can be constructed if we allow non-integral values for the charges. We can dispense entirely with the basic baryon $b$ if we assign to the triplet $t$ the following properties: spin $\frac{1}{2}$, $z = -\frac{1}{3}$, and baryon number $\frac{1}{3}$. We then refer to the members $u^3_2$, $d^{-1}_2$, and $s^{-\frac{1}{3}}$ of the triplet as "quarks" and the members of the anti-triplet as anti-quarks $\bar{q}$. Baryons can now be constructed from quarks by using the combinations $(q q q)$, $(q q q q q)$, etc., while mesons are made out of $(q \bar{q})$, $(q q \bar{q} q)$, etc. It is assuming that the lowest baryon configuration $(q q q)$ gives just the representations $1$, $8$, and $10$ that have been observed, while the lowest meson configuration $(q \bar{q})$ similarly gives just $1$ and $8$.

6) James Joyce, Finnegans Wake: „three quarks for Muster Mark“
Physical motivation

Evidence for substructure of hadrons

Extension (form factors, e.g., root-mean-square charge radius of the proton $r_p^E = (0.8751 \pm 0.0061)$ fm)

Excitation spectrum

Deep inelastic scattering (pointlike partons)

Interpretation: Hadrons are (complicated) bound states of fundamental degrees of freedom

Fundamental Theory: Quantum chromodynamics (QCD)

QCD is a non-Abelian gauge theory with gauge group $G = SU(3)$

Matter fields of QCD (quarks) are fermions with spin 1/2, which show up in six different flavors...
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### Light quarks

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<th>flavor</th>
<th>$u$</th>
<th>$d$</th>
<th>$s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>mass [MeV]</td>
<td>$2.2^{+0.6}_{-0.4}$</td>
<td>$4.7^{+0.5}_{-0.4}$</td>
<td>$96^{+8}_{-4}$</td>
</tr>
<tr>
<td>charge [$e &gt; 0$]</td>
<td>$\frac{2}{3}$</td>
<td>$-\frac{1}{3}$</td>
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</tr>
<tr>
<td>$I_3$</td>
<td>$+\frac{1}{2}$</td>
<td>$-\frac{1}{2}$</td>
<td>0</td>
</tr>
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strangeness: $-1$
### Heavy quarks

<table>
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<tr>
<th>flavor</th>
<th>$c$</th>
<th>$b$</th>
<th>$t$</th>
</tr>
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<tr>
<td>mass [GeV]</td>
<td>$1.28 \pm 0.03$</td>
<td>$4.18^{+0.04}_{-0.03}$</td>
<td>$173.1 \pm 0.6$</td>
</tr>
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<tr>
<td>$l_3$</td>
<td>$0$</td>
<td>$0$</td>
<td>$0$</td>
</tr>
<tr>
<td></td>
<td>charm: +1</td>
<td>bottom: $-1$</td>
<td>top: +1</td>
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See [http://pdg.lbl.gov](http://pdg.lbl.gov)
Each quark flavor comes with three colors
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Motivation

\[ \Delta^{++} (S_z = \frac{3}{2}) = u \uparrow u \uparrow u \uparrow \]
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- **Motivation**
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  \Delta^{++}(S_z = \frac{3}{2}) = u \uparrow u \uparrow u \uparrow
  \]

- **Contradiction to Pauli principle**
- **Solution: Slater determinant**

\[
\frac{1}{\sqrt{6}} \begin{vmatrix}
  r_1 & g_1 & b_1 \\
  r_2 & g_2 & b_2 \\
  r_3 & g_3 & b_3
\end{vmatrix}
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- General \( N_c \):
  \[
  \frac{1}{\sqrt{N_c!}} \epsilon_{i_1 \ldots i_{N_c}} \chi^{i_1} \otimes \ldots \otimes \chi^{i_{N_c}}
  \]

Stefan Scherer

Symmetries in Physics: Introduction and Overview
To each quark $q$ there exists an antiquark $\bar{q}$ with

- the same mass, $m_q = m_{\bar{q}}$, etc.
- opposite charge, $Q_{\bar{q}} = -2/3$, etc.
- opposite internal quantum numbers, $I_3 = -1/2$ for $\bar{u}$, etc., $S = +1$ for $\bar{s}$, etc.
- opposite color quantum numbers, $r_{\bar{u}}$ vs. $\bar{r}_u$, etc.

8 gluons (spin 1, massless) mediate the interaction

The interaction is flavor independent

Free quarks and gluons have not been observed ($\Rightarrow$ color-confinement hypothesis)

Baryons: $qqq$ states; color neutral via Slater determinant

Mesons: $q\bar{q}$ (quark-antiquark) states; color neutral via

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