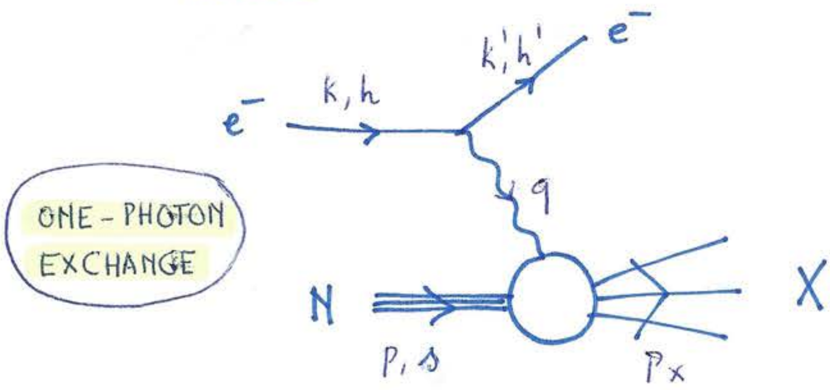
DEEP INELASTIC LEPTON-NUCLEON SCATTERING

PARTON MODEL

⇒ ELECTRON-NUCLEON SCATTERING : KINEMATICS



$h\,(h')$: INITIAL (FINAL) e^- HELICITIES

$h = \pm \frac{1}{2}$, $h' = \pm \frac{1}{2}$

s : INITIAL NUCLEON SPIN

$s = \pm \frac{1}{2}$

$$\left.\begin{array}{ll} k\ (E_e, \bar{k}) & E_e = |\bar{k}| \\ k'\ (E_e', \bar{k}') & E_e' = |\bar{k}'| \end{array}\right\} m_e \approx 0$$

LAB SYSTEM $\quad p\ (M, 0)$

$$q \equiv k - k' = (\nu, \bar{q})$$

$$q^2 = \nu^2 - \bar{q}^2$$

$$= -2k.k' = -4E_eE_e' \sin^2\frac{\theta}{2} < 0 \quad \text{SPACELIKE } \gamma$$

$$-q^2 \equiv +Q^2 > 0$$

⇒ ELECTRON - NUCLEON SCATTERING : CROSS SECTION

IN LAB SYSTEM

$$d\sigma = \frac{1}{v_{rel}} \cdot \frac{1}{(2E_e)(2M)} \sum_{h'} \frac{d^3\bar{k}'}{(2\pi)^3\, 2E'_e} \sum_X (2\pi)^4\, \delta^4(q+P-P_X)$$

SUM OVER FINAL LEPTON HELICITY

$$\cdot \left| \bar{U}(k'h')\,\gamma_\nu\, U(k\,h) \cdot \frac{e^2}{Q^2} \cdot \langle X | J^\nu(0) | P\, s\rangle \right|^2$$

$$\downarrow \qquad v_{rel} = \frac{|\bar{k}|}{E_e} + \frac{|\bar{P}|}{E_N} = \frac{|\bar{k}|}{E_e} \approx 1 \qquad \text{(INITIAL FLUX)}$$

$$d\sigma = \frac{(4\pi\alpha)^2}{Q^4} \cdot \frac{1}{4ME_e} \frac{d\Omega'_e \cdot dE'_e\, E'_e}{(2\pi)^3\; 2} \cdot$$

$$\alpha \equiv \frac{e^2}{4\pi} \simeq \frac{1}{137}$$

$$\cdot \sum_{h'} \bar{U}(k\,h)\,\gamma_\mu\, U(k'h')\, \bar{U}(k'h')\,\gamma_\nu\, U(k\,h)$$

$$\cdot \sum_X (2\pi)^4\,\delta^4(q+P-P_X) \cdot \langle P s | J^{\mu+}(0) | X\rangle \langle X | J^\nu(0) | P s\rangle$$

$$\downarrow$$

$$\boxed{\left(\frac{d\sigma}{d\Omega'_e\, dE'_e}\right)^{LAB} = \frac{\alpha^2}{Q^4}\, \frac{1}{2M} \cdot \frac{E'_e}{E_e} \cdot L_{\mu\nu} \cdot W^{\mu\nu}}$$

LEPTON TENSOR $\quad L_{\mu\nu} \equiv \sum_{h'} \bar{U}(k\,h)\,\gamma_\mu\, U(k'h')\,\bar{U}(k'h')\,\gamma_\nu\, U(k\,h)$

HADRON TENSOR $\quad W^{\mu\nu} \equiv \frac{1}{2\pi} \sum_X (2\pi)^4\,\delta^4(q+P-P_X) \cdot \langle P s | J^{\mu+}(0) | X\rangle \langle X | J^\nu(0) | P s\rangle$

- LEPTON TENSOR

$$L_{\mu\nu} \equiv \sum_{h'} \bar{u}(k\,h)\,\gamma_\mu\, u(k'h')\,\bar{u}(k'h')\,\gamma_\nu\, u(k\,h)$$

$$\Big\downarrow \quad \sum_{h'} u(k'h')\,\bar{u}(k'h') = \not{k}'$$

$$= \mathrm{Tr}\left\{\gamma_\mu \not{k}' \gamma_\nu\, u(k\,h)\,\bar{u}(k\,h)\right\}$$

$$u(k\,h)\,\bar{u}(k\,h)$$

$$= \underbrace{\left(\frac{1+(2h)\gamma_5}{2}\right)}_{\text{HELICITY PROJECTOR}} \sum_{h''} u(k\,h'')\,\bar{u}(k\,h'') = \left(\frac{1+2h\,\gamma_5}{2}\right)\not{k}$$

$$L_{\mu\nu} = \mathrm{Tr}\left\{\gamma_\mu \not{k}' \gamma_\nu \left(\frac{1+(2h)\gamma_5}{2}\right)\not{k}\right\}$$

$$= \frac{1}{2}\underbrace{\mathrm{Tr}\left\{\gamma_\mu \not{k}' \gamma_\nu \not{k}\right\}}_{4\{k'_\mu k_\nu + k'_\nu k_\mu - k.k' g_{\mu\nu}\}} - \frac{(2h)}{2}\underbrace{\mathrm{Tr}\left\{\gamma_5\, \gamma_\mu \not{k}' \gamma_\nu \not{k}\right\}}_{4i\,\varepsilon_{\mu\beta\nu\alpha}\, k'^\beta k^\alpha \,=\, -4i\,\varepsilon_{\mu\nu\alpha\beta}\, k^\alpha q^\beta}$$

$$(\varepsilon_{0123} = +1)$$

$$\boxed{L_{\mu\nu} = L^S_{\mu\nu} + i\, L^A_{\mu\nu}}$$

SYMMETRIC PART: $L^S_{\mu\nu} = 2\left\{k_\mu k'_\nu + k_\nu k'_\mu - k.k'\, g_{\mu\nu}\right\}$

ANTI-SYMMETRIC PART: $L^A_{\mu\nu} = 2(2h)\,\varepsilon_{\mu\nu\alpha\beta}\, k^\alpha q^\beta$

• HADRON TENSOR

$$W^{\mu\nu} \equiv \frac{1}{2\pi} \sum_X (2\pi)^4 \delta^4(q + p - p_X)$$
$$\cdot \langle p s | J^{\mu +}(0) | X \rangle \langle X | J^{\nu}(0) | p s \rangle$$

$W^{\mu\nu}$ CAN BE PARAMETRIZED IN TERMS OF 4 STRUCTURE FUNCTIONS

$$\boxed{W^{\mu\nu} = W_S^{\mu\nu} + i\, W_A^{\mu\nu}}$$

SYMMETRIC i.e. SPIN INDEPENDENT

ANTI-SYMMETRIC (i.e. SPIN DEPENDENT)

$\Rightarrow$ $W_S^{\mu\nu}$ 2 INDEPENDENT 4 MOMENTA q^μ, p^μ

TENSORS $q^\mu q^\nu,\ p^\mu p^\nu,\ q^\mu p^\nu,\ q^\nu p^\mu,\ g^{\mu\nu}$

+ GAUGE INVARIANCE $q_\mu W^{\mu\nu} = q_\nu W^{\mu\nu} = 0$

$\Downarrow$

ONLY 2 TENSORS SURVIVE

$$\left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2} \right) \text{ AND } \left(p^\mu - \frac{p.q}{q^2} q^\mu \right)\left(p^\nu - \frac{p.q}{q^2} q^\nu \right)$$

$$\boxed{\begin{aligned} \frac{1}{2M} W_S^{\mu\nu} \equiv & \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2} \right) W_1(Q^2, \nu) \\ & + \frac{1}{M^2} \left(p^\mu - \frac{p.q}{q^2} q^\mu \right)\left(p^\nu - \frac{p.q}{q^2} q^\nu \right) W_2(Q^2, \nu) \end{aligned}}$$

W_1 & W_2 → DEPEND ON Q^2, ν

→ UNPOLARIZED NUCLEON STRUCTURE FUNCTIONS

⇒ $W_A^{\mu\nu}$ DEPENDS LINEARLY ON NUCLEON SPIN

S^μ: 4-VECTOR : NUCLEON POLARIZATION VECTOR

IN NUCLEON REST FRAME $S^\mu = (0, \bar{n})$

$\bar{n}^2 = 1$

$\bar{n}$: AXIS ALONG WHICH NUCLEON SPIN IS QUANTIZED

$$\begin{cases} S_\mu S^\mu = -\bar{n}^2 = -1 \\ S_\mu \cdot p^\mu = 0 . M - \bar{n} . \bar{0} = 0 \end{cases}$$

LORENTZ INV. (HOLD IN ANY FRAME)

$$\underline{\underline{S_\mu S^\mu = -1 \quad , \quad S_\mu p^\mu = 0}}$$

$W_A^{\mu\nu}$ → DEPENDS LINEARLY ON S

→ SATISFIES $q_\mu W_A^{\mu\nu} = q_\nu W_A^{\mu\nu} = 0$

$W_A^{\mu\nu}$ INVOLVES → $\varepsilon^{\mu\nu\alpha\beta} q_\alpha S_\beta$

→ $\varepsilon^{\mu\nu\alpha\beta} q_\alpha p_\beta . (S . q)$

OR INSTEAD OF

$$\varepsilon^{\mu\nu\alpha\beta}\, q_\alpha\, P_\beta\, (S.q)$$

WE CAN USE

$$\varepsilon^{\mu\nu\alpha\beta}\, q_\alpha\, \underbrace{\left(S_\beta - \frac{S.q}{P.q} P_\beta \right)}_{\text{'TRANSVERSE' SPIN VECTOR}}$$

$$\equiv (S_\perp)_\beta \qquad \left(q_\beta \cdot S_\perp^\beta = 0 \right)$$

$$\boxed{\frac{1}{2M} W_A^{\mu\nu} \equiv \varepsilon^{\mu\nu\alpha\beta}\, q_\alpha \left\{ M\, S_\beta\, G_1(Q^2,\nu) + \frac{P.q}{M}\left[S_\beta - \frac{S.q}{P.q} P_\beta \right] G_2(Q^2,\nu) \right\}}$$

G_1, G_2 : SPIN - DEPENDENT
NUCLEON STRUCTURE FUNCTIONS

- 'SCALING' FUNCTIONS

INSTEAD OF ν : USE $\boxed{x_B \equiv \frac{Q^2}{2M\nu}}$ BJORKEN SCALING VARIABLE

$$\boxed{\begin{aligned} M W_1 &\equiv F_1(Q^2, x_B) \\ \nu W_2 &\equiv F_2(Q^2, x_B) \\ M^2 \nu G_1 &\equiv g_1(Q^2, x_B) \\ M \nu^2 G_2 &\equiv g_2(Q^2, x_B) \end{aligned}}$$

$\hookrightarrow$ F_1, F_2, g_1, g_2 ARE DIMENSIONLESS

$$\boxed{\begin{aligned} W_S^{\mu\nu} = 2 \Big\{ & \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) F_1 \\ & + \frac{1}{p.q} \left(p^\mu - \frac{p.q}{q^2} q^\mu\right)\left(p^\nu - \frac{p.q}{q^2} q^\nu\right) F_2 \Big\} \end{aligned}}$$

$$\boxed{\begin{aligned} W_A^{\mu\nu} = 2\, \varepsilon^{\mu\nu\alpha\beta} q_\alpha \frac{1}{\nu} \Big\{ & S_\beta\, g_1 \\ & + \left(S_\beta - \frac{S.q}{p.q} p_\beta\right) g_2 \Big\} \end{aligned}}$$

- UNPOLARIZED CROSS SECTION

$$\left(\frac{d\sigma}{d\Omega_e' \, dE_e'}\right)^{LAB} = \frac{\alpha^2}{Q^4} \cdot \frac{E_e'}{E_e} \cdot \frac{1}{2M} \cdot L^S_{\mu\nu} \cdot W_S^{\mu\nu}$$

$$\Rightarrow L^S_{\mu\nu} \cdot H^{\mu\nu}_S = 2 \left\{ k_\mu k'_\nu + k_\nu k'_\mu - k.k' \, g_{\mu\nu} \right\}$$

$$\cdot \, 2 \left\{ \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2} \right) F_1 \right.$$

$$\left. + \frac{1}{P.q} \left(P^\mu - \frac{P.q}{q^2} q^\mu \right) \left(P^\nu - \frac{P.q}{q^2} q^\nu \right) F_2 \right\}$$

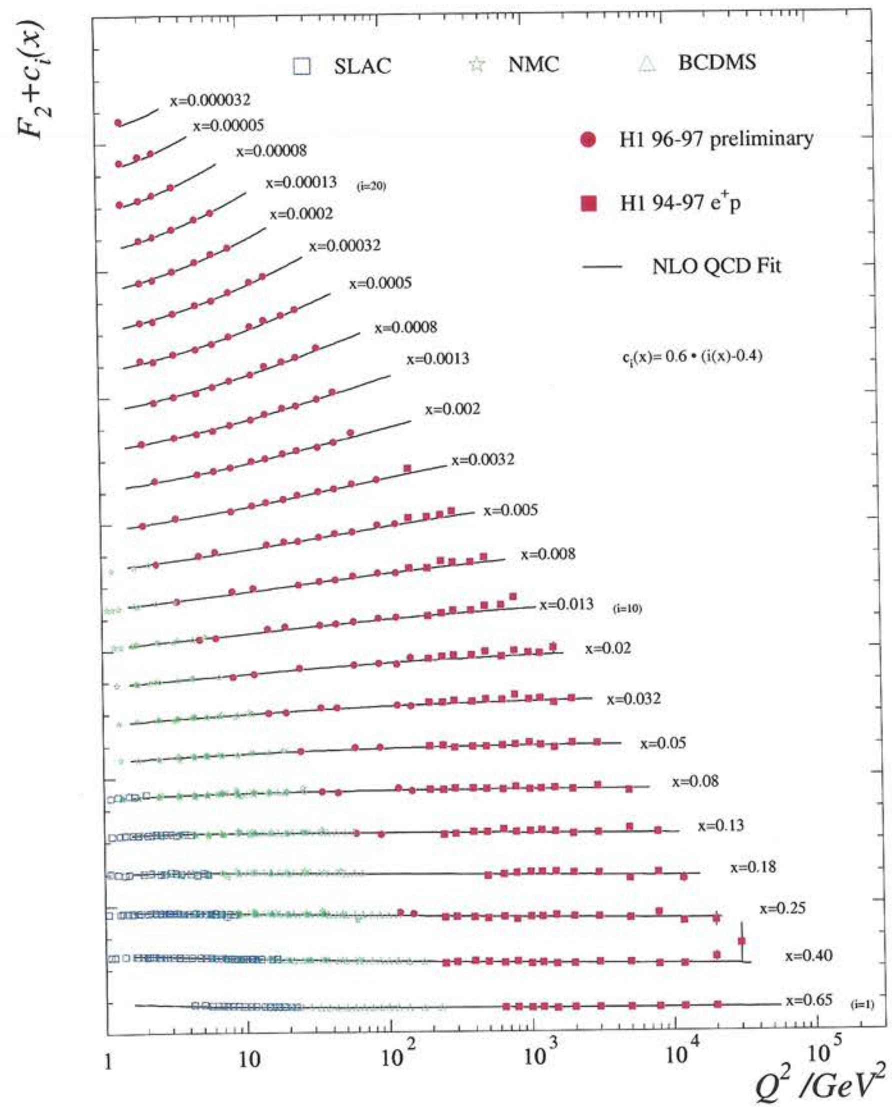
$$= 4 \left\{ 2 \, k.k' \, F_1 + \frac{1}{P.q} \left(2 (P.k)(P.k') - k.k' \, M^2 \right) F_2 \right\}$$

$$= 4 \left\{ Q^2 F_1 + \frac{M^2}{P.q} E_e E_e' \left(1 + \cos\theta \right) F_2 \right\}$$

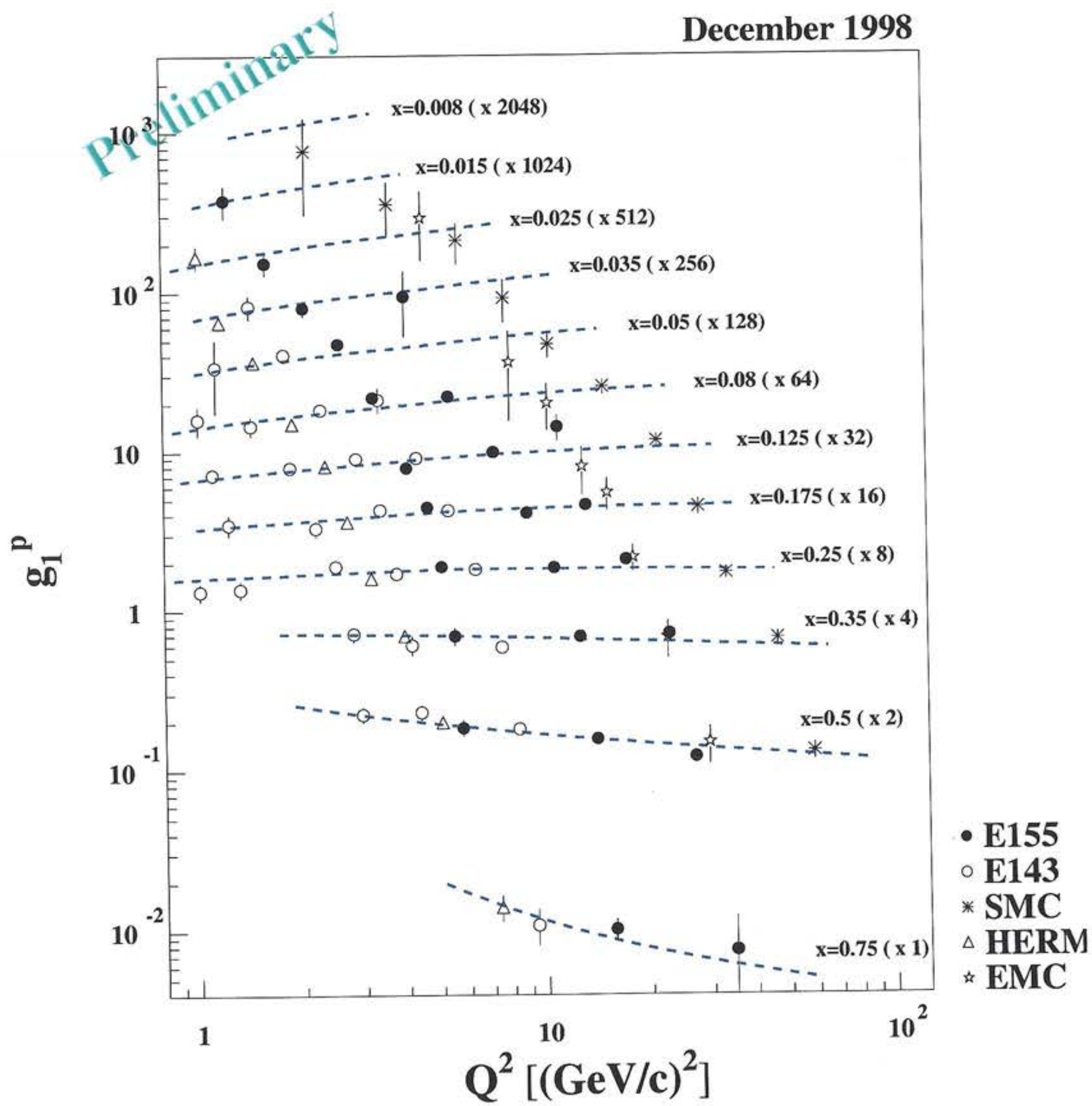
$$= 8 \, E_e E_e' \left\{ 2 \sin^2 \theta/2 \, F_1 + \frac{M}{\nu} \cos^2 \theta/2 \, F_2 \right\}$$

$$\boxed{\left(\frac{d\sigma}{d\Omega_e' \, dE_e'}\right)^{LAB}_{UNPOL} = \frac{\alpha^2}{Q^4} \, \frac{4 E_e'^2}{M} \left\{ 2 \sin^2 \theta/2 \, F_1 + \frac{M}{\nu} \cos^2 \theta/2 \, F_2 \right\}}$$

World data on F_1^p



World data on g_1^p



- BJORKEN SCALING

IN LIMIT $Q^2 \gg$

$\nu \gg$

$$x_B = \frac{Q^2}{2M\nu} = \text{CONSTANT}$$

$$F_1(x_B, Q^2) \longrightarrow F_1(x_B)$$

$$F_2(x_B, Q^2) \longrightarrow F_2(x_B)$$

AND $\Big\}$ $F_2 = 2x_B F_1$

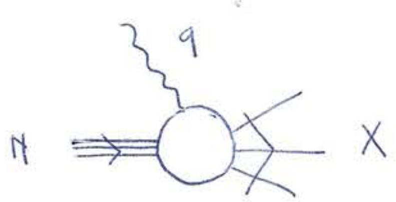
(CALLAN - GROSS RELATION)

$$g_1(x_B, Q^2) \longrightarrow g_1(x_B)$$

$$g_2(x_B, Q^2) \longrightarrow g_2(x_B)$$

⇒ HADRONIC TENSOR IN PARTON MODEL

- $$W^{\mu\nu} = \frac{1}{2\pi} \sum_X (2\pi)^4 \delta^4(q + p - p_X) \langle N | J^{\mu\dagger}(0) | X \rangle \langle X | J^\nu(0) | N \rangle$$



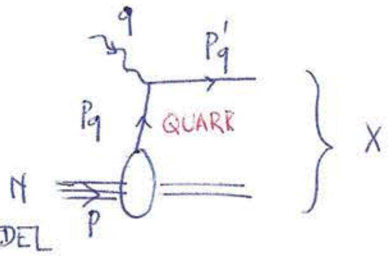
$\approx$

IN PARTON MODEL

$Q^2 \gg$

$\nu \gg$

$x_B = \frac{Q^2}{2M\nu}$ = CONSTANT



EVALUATE TENSOR IN FRAME WHERE PROTON MOVES WITH LARGE MOMENTUM

(INFINITE MOMENTUM FRAME)

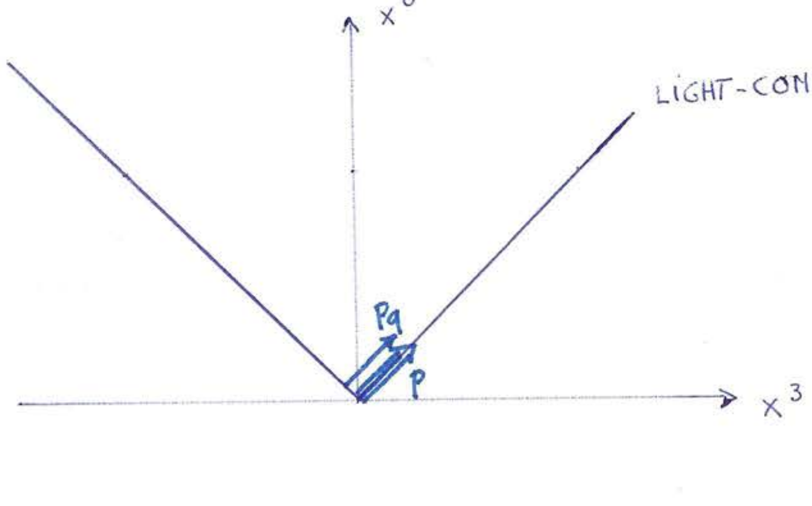
↳ THE QUARKS MOVE NEARLY COLLINEAR WITH PROTON $\boxed{p_q \approx x\, p}$

(i.e. NEGLECT THEIR SMALL TRANSVERSE MOMENTUM $\vec{p}_{q\perp}$) $p_q^2 \approx 0$

$$\langle \vec{p}_{q\perp}^{\,2} \rangle \approx (0.3 \text{ GeV})^2 \ll Q^2$$

(QUARKS ARE NEARLY MASSLESS)

SPACE - TIME PICTURE



IN PARTON MODEL (INCOHERENT SCATTERING OFF INDIVIDUAL QUARKS)

$$\boxed{W^{\mu\nu}_{\substack{\text{PARTON}\\ \text{MODEL}}} = \sum_q \sum_s \int_0^1 \frac{dx}{x} \; \underbrace{n_q(x, s)}_{} \cdot \underbrace{e_q^2 \, w^{\mu\nu}}_{}}$$

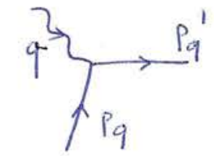
SUM OVER QUARK SPECIES (FLAVORS)

SUM OVER INITIAL QUARK SPIN PROJECTIONS

INITIAL FLUX
$E_q = x \, E_N$

NUMBER DENSITY OF QUARKS WITH MOMENTUM FRACTION x & SPIN PROJECTION $s = \pm \frac{1}{2}$

SCATTERING OFF A SINGLE QUARK



- SCATTERING OFF AN INDIVIDUAL QUARK : $w^{\mu\nu}$

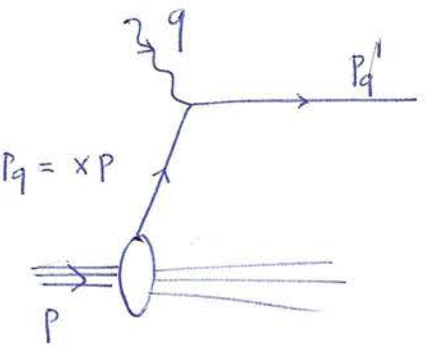
$$w^{\mu\nu} = \frac{1}{2\pi} \underbrace{\sum_{s'} \int \frac{d^3 \vec{P}_q'}{(2\pi)^3 \, 2E_q'}}_{\sum_X} (2\pi)^4 \, \delta^4 (q + P_q - P_q')$$

$$\cdot \, \bar{u}(P_q \, s) \, \gamma^\mu \, u(P_q' \, s') \, \bar{u}(P_q' \, s') \, \gamma^\nu \, u(P_q \, s)$$

$$= \underbrace{\frac{1}{2E_q'} \, \delta(P_q'^0 - E_q')}_{\substack{\text{III} \\ \delta(P_q'^2 - m_q^2) \\ \text{॥} \\ 0}} \; (P_q'^0 > 0) \quad . \quad \text{Tr} \left\{ \left(\frac{1 + (2s)\gamma_5}{2} \right) \not{P}_q \, \gamma^\mu \, \not{P}_q' \, \gamma^\nu \right\}$$

$$\downarrow \quad \delta(P_q'^2) = \delta\big((q + \overset{P_q = xP}{\overbrace{xP}})^2 \big) = \delta(x \, 2P.q - Q^2)$$

$$= \frac{1}{2P.q} \, \delta\left(x - \frac{Q^2}{2P.q}\right)$$



$$\delta\left(p_q'^2\right) = \frac{1}{Q^2}\left(\frac{Q^2}{2p\cdot q}\right)\delta\left(x - \frac{Q^2}{2p\cdot q}\right)$$

$\Downarrow$ USING $x_B \equiv \dfrac{Q^2}{2p\cdot q}$ $\left(= \dfrac{Q^2}{2M\nu}\right.$ IN LAB SYSTEM$\left.\right)$

$$\delta\left(p_q'^2\right) = \frac{x_B}{Q^2}\,\delta\left(x - x_B\right)$$

$\Downarrow$

PHOTON SCATTERS OFF QUARK WHICH HAS MOMENTUM FRACTION x_B OF PROTON MOMENTUM

$$\therefore \quad w^{\mu\nu} = \frac{x_B}{Q^2}\,\delta\left(x - x_B\right) \cdot \frac{1}{2}\left\{ \mathrm{Tr}\left\{ \not{p}_q \gamma^\mu \not{p}_q' \gamma^\nu \right\} + (2s)\,\mathrm{Tr}\left\{ \gamma_5 \not{p}_q \gamma^\mu \not{p}_q' \gamma^\nu \right\} \right\}$$

$$w^{\mu\nu} = \frac{x_B}{Q^2}\,\delta\left(x - x_B\right) \cdot 2\left\{ p_q^\mu p_q'^\nu + p_q^\nu p_q'^\mu - \left(p_q \cdot p_q'\right) g^{\mu\nu} + (2s)\, i\, \varepsilon^{\mu\nu\alpha\beta}\, p'_{q\alpha}\, p_{q\beta} \right\}$$

$$\downarrow \qquad P_q = x P$$

$$P_q' = q + x P$$

$$w^{\mu\nu} = \frac{x_B}{Q^2} \delta(x - x_B)$$

$$\cdot 2 \cdot \Big\{ x_B^2 \Big[P^\mu \Big(P^\nu + \frac{1}{x_B} q^\nu \Big) + P^\nu \Big(P^\mu + \frac{1}{x_B} q^\mu \Big) \Big]$$

$$- \frac{Q^2}{2} g^{\mu\nu} + (2s)\, i\, \varepsilon^{\mu\nu\alpha\beta} x_B \, q_\alpha P_\beta \Big\}$$

$$\downarrow$$

$$w^{\mu\nu} = \frac{x_B}{Q^2} \delta(x - x_B)$$

$$\cdot 2 \cdot \Big\{ 2 x_B^2 \Big(P^\mu - \frac{P \cdot q}{q^2} q^\mu \Big) \Big(P^\nu - \frac{P \cdot q}{q^2} q^\nu \Big)$$

$$+ \frac{Q^2}{2} \Big(- g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2} \Big)$$

$$+ x_B (2s)\, i\, \varepsilon^{\mu\nu\alpha\beta} q_\alpha P_\beta \Big\}$$

$$\boxed{\begin{aligned} W^{\mu\nu}_{\substack{\text{PARTON}\\ \text{MODEL}}} = & \sum_q e_q^2 \sum_s n_q(x_B, s) \cdot \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) \\ & + \sum_q e_q^2 \sum_s n_q(x_B, s) \cdot \frac{4x_B^2}{Q^2} \left(P^\mu - \frac{P.q}{q^2} q^\mu\right)\left(P^\nu - \frac{P.q}{q^2} q^\nu\right) \\ & + \sum_q e_q^2 \sum_s (2s)\, n_q(x_B, s) \cdot \frac{2x_B}{Q^2}\, i\, \varepsilon^{\mu\nu\alpha\beta} q_\alpha P_\beta \end{aligned}}$$

COMPARE THIS WITH GENERAL EXPRESSION

$$\begin{aligned} W^{\mu\nu} = \; 2 \Big\{ & \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) F_1 \\ & + \frac{1}{P.q} \left(P^\mu - \frac{P.q}{q^2} q^\mu\right)\left(P^\nu - \frac{P.q}{q^2} q^\nu\right) F_2 \\ & + i\, \varepsilon^{\mu\nu\alpha\beta} q_\alpha \frac{M}{P.q} \left[S_\beta\, g_1 + S_{\perp\beta}\, g_2 \right] \Big\} \end{aligned}$$

- UNPOLARIZED STRUCTURE FUNCTIONS

IN PARTON MODEL

$$F_1(x_B, Q^2) = \frac{1}{2} \sum_q e_q^2 \sum_s n_q(x_B, s)$$ FUNCTION OF x_B ONLY!

$$F_2(x_B, Q^2) = x_B \sum_q e_q^2 \sum_s n_q(x_B, s)$$ FUNCTION OF x_B ONLY!

$\Downarrow$

BJORKEN SCALING

$$\sum_s n_q(x_B, s) = n_q(x_B, +\tfrac{1}{2}) + n_q(x_B, -\tfrac{1}{2})$$

$$\equiv q(x_B)$$ UNPOLARIZED QUARK DISTRIBUTION IN NUCLEON

FOR EACH FLAVOR

$u(x_B),\ d(x_B),\ s(x_B),\ \ldots$ QUARK NUMBER DENSITIES IN NUCLEON

IN PARTON MODEL

$$\boxed{F_1(x_B, \cancel{Q^2}) = \frac{1}{2}\sum_q e_q^2 \left\{ q(x_B) + \bar{q}(x_B) \right\}}$$

↳ ANTI-QUARK CONTRIBUTION

$$= \frac{1}{2}\left\{ \frac{4}{9}u(x_B) + \frac{1}{9}d(x_B) + \frac{1}{9}s(x_B) + \cdots + \frac{4}{9}\bar{u}(x_B) + \frac{1}{9}\bar{d}(x_B) + \frac{1}{9}\bar{s}(x_B) + \cdots \right\}$$

$$\boxed{F_2(x_B, \cancel{Q^2}) = \sum_q e_q^2\, \underbrace{x_B\left\{ q(x_B)\right.}\left.+ \bar{q}(x_B) \right\}}$$

↳ QUARK MOMENTUM DENSITIES

$$F_2(x_B, \cancel{Q^2}) = 2x_B\, F_1(x_B, \cancel{Q^2})$$: CALLAN - GROSS RELATION

↑ IN PARTON MODEL

IS A CONSEQUENCE OF SPIN 1/2 NATURE OF PARTONS

↓

QUARKS ARE DIRAC PARTICLES

- SPIN DEPENDENT STRUCTURE FUNCTIONS

NUCLEON MOVING WITH HIGH VELOCITY

↳ SPIN ALIGNED ALONG MOMENTUM (POSITIVE HELICITY)

$$S^\beta \approx \frac{p^\beta}{M} \qquad S \cdot p = M \approx 0$$

$(E_N \gg M)$

$S_\perp^\beta \approx 0 \quad \Rightarrow$ IN PARTON MODEL

$g_2(x_B, Q^2)$ CANNOT BE ACCESSED

$$g_1(x_B, Q^2) = \frac{1}{2} \sum_q e_q^2 \sum_s (2s)\, n_q(x_B, s)$$

$$\sum_s (2s)\, n_q(x_B, s) = n_q\left(x_B, +\tfrac{1}{2}\right) - n_q\left(x_B, -\tfrac{1}{2}\right)$$

$$\equiv \Delta q(x_B)$$

QUARK HELICITY DISTRIBUTION IN NUCLEON

$$\boxed{g_1(x_B, \cancel{Q^2}) = \frac{1}{2} \sum_q e_q^2 \left\{ \Delta q(x_B) + \Delta \bar{q}(x_B) \right\}}$$ BJORKEN SCALING

$$= \frac{1}{2} \left\{ \frac{4}{9} \Delta u(x_B) + \frac{1}{9} \Delta d(x_B) + \frac{1}{9} \Delta s(x_B) + \cdots \right.$$

$$\left. + \frac{4}{9} \Delta \bar{u}(x_B) + \frac{1}{9} \Delta \bar{d}(x_B) + \frac{1}{9} \Delta \bar{s}(x_B) + \cdots \right\}$$

⇒ VALENCE & SEA-QUARK DISTRIBUTIONS

$q(x_B)$: QUARK DISTRIBUTION IN PROTON

$\bar{q}(x_B)$: ANTI-QUARK DISTRIBUTION IN PROTON

- "VALENCE" DISTRIBUTION

$$q_V(x_B) \equiv q(x_B) - \bar{q}(x_B)$$

- "SEA" DISTRIBUTION

$$q_S(x_B) \equiv \bar{q}(x_B)$$

$$\rightsquigarrow \boxed{q(x_B) = q_V(x_B) + q_S(x_B)}$$

⇒ UNPOLARIZED SUM RULES

- PROTON : $|p\rangle = c_1 |uud\rangle + c_2 |uud\,u\bar{u}\rangle + c_3 |uud\,d\bar{d}\rangle + \dots$

$$\int_0^1 dx \; u_V(x) = 2$$

$$\int_0^1 dx \; d_V(x) = 1$$

$$\int_0^1 dx \left[s(x) - \bar{s}(x) \right] = 0$$

NET STRANGENESS OF PROTON $= 0$

- NEUTRON :

q^p : QUARK DISTR IN PROTON

q^n : QUARK DISTR IN NEUTRON

$$|n\rangle = c_1\,|ddu\rangle + c_2\,|ddu\,d\bar{d}\rangle + c_3\,|ddu\,u\bar{u}\rangle + \cdots$$

↳ OBTAINED FROM $|p\rangle$ BY $u \leftrightarrow d$

$$u^n(x) = d^p(x) \equiv d(x)$$

SU(2) SYMMETRY

$$d^n(x) = u^p(x) \equiv u(x)$$

$$\int_0^1 dx\; d_v^n(x) = \int_0^1 dx\; u_v(x) = 2$$

$$\int_0^1 dx\; u_v^n(x) = \int_0^1 dx\; d_v(x) = 1$$

$$F_1^p = \frac{1}{2}\left\{\frac{4}{9}(u+\bar{u}) + \frac{1}{9}(d+\bar{d}) + \frac{1}{9}(s+\bar{s})\right\}$$

$$F_1^n = \frac{1}{2}\left\{\frac{4}{9}(d+\bar{d}) + \frac{1}{9}(u+\bar{u}) + \frac{1}{9}(s+\bar{s})\right\}$$

∴ BY DOING EXPERIMENTS ON BOTH PROTON & NEUTRON

$\Longrightarrow$ u, d QUARK FLAVOR SEPARATION

⇒ SEPARATION OF QUARK & ANTI-QUARK DISTRIBUTIONS

HOW CAN WE SEPARATE q & $\bar{q}$?

↳ EM PROBE COUPLES IN SAME WAY TO q & $\bar{q}$

$$F_1 = \frac{1}{2} e_q^2 (q + \bar{q})$$

↳ NEED PROBE WHICH COUPLES DIFFERENTLY TO q & $\bar{q}$

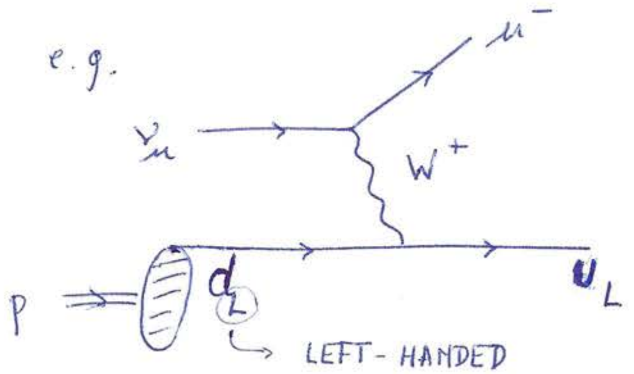
MASSLESS q : LEFT-HANDED

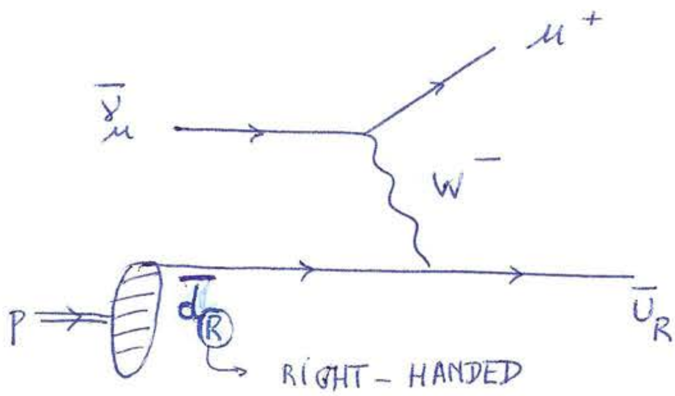
MASSLESS $\bar{q}$: RIGHT-HANDED

⇓

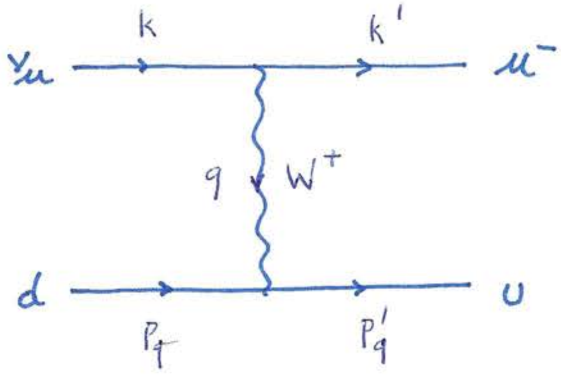
WEAK INTERACTION COUPLES DIFFERENTLY TO q & $\bar{q}$

e.g.





⇒ NEUTRINO – QUARK SCATTERING



$$(-i)\,\frac{g_W}{2\sqrt{2}}\,\gamma_\alpha(1-\gamma_5)$$

$$-\frac{i\,g^{\alpha\beta}}{q^2-m_W^2}$$

$$(-i)\,\frac{g_W}{2\sqrt{2}}\,\gamma_\beta(1-\gamma_5)$$

(NEGLECT $d \leftrightarrow s$ MIXING)

WITH $\boxed{\dfrac{g_W^2}{8m_W^2} \equiv \dfrac{G_F}{\sqrt{2}}}$ → FERMI WEAK COUPLING CONSTANT

FOR $-q^2 \ll m_W^2$ ⇝ NEGLECT q^2 IN W PROPAGATOR

- $$\mathcal{M} = -i\left(\frac{G_F}{\sqrt{2}}\right)\cdot \bar{u}_{\mu^-}\gamma_\nu(1-\gamma_5)u_{\nu_\mu}\cdot \bar{u}_u\gamma^\nu(1-\gamma_5)u_d$$

- CROSS SECTION

$$d\sigma = \frac{1}{2E_k\,2E_{p_q}\,v_{rel}}\;\frac{d^3\vec{k}'}{(2\pi)^3\,2E_k'}\;\frac{d^3\vec{p}_q'}{(2\pi)^3\,2E_{p_q}'}\cdot(2\pi)^4\,\delta^4(k+p_q-k'-p_q')\cdot|\mathcal{M}|^2$$

↓ NEGLECT MASSES OF ALL PARTICLES

+ CONSIDER C.m. SYSTEM $\quad |\vec{k}| = |\vec{p}_q| = |\vec{k}'| = |\vec{p}_q'| = \dfrac{\sqrt{s}}{2}$

$$\left.\begin{aligned}\hat{s} &= (k+p_q)^2\\ \hat{u} &= (k'-p_q)^2\\ \hat{t} &= (k-k')^2 = q^2 = -Q^2\end{aligned}\right\}\quad \hat{s}+\hat{u} = Q^2$$

$$\hookrightarrow \quad 2E_k \, 2E_{p_q} \, v_{rel} = 4 E_k E_{p_q} \left(\frac{|\bar{k}|}{E_k} + \frac{|\bar{k}|}{E_{p_q}} \right)$$

$$= 4 |\bar{k}| \sqrt{\hat{s}} = 2\hat{s}$$

$$\hookrightarrow \quad \hat{t} = -2 k.k' = -2|\bar{k}|^2 (1 - \cos\Theta_{cm})$$

$$d\hat{t} = +2 |\bar{k}|^2 \, d\cos\Theta_{cm}$$

$$\hookrightarrow \quad \int d\phi \rightsquigarrow 2\pi \qquad\qquad \hookrightarrow \delta\left(\sqrt{\hat{s}} - 2|\bar{k}'|\right) = \frac{1}{2}\delta\left(|\bar{k}'| - \frac{\sqrt{\hat{s}}}{2}\right)$$

$$\frac{d\sigma}{d\hat{t}} = \underbrace{\frac{1}{2\hat{s}}}_{\frac{1}{4E_k E_{p_q} v_{rel}}} \cdot \underbrace{\frac{1}{2\pi}}_{\text{FROM } \int d\phi} \cdot \underbrace{\frac{1}{2\hat{s}}}_{\frac{1}{2 \cdot 4 E_{k'} E'_{p_q}}} \cdot \underbrace{\frac{1}{2}}_{\text{FROM } \delta} \cdot |\mathcal{M}|^2$$

$$\boxed{\frac{d\sigma}{d\hat{t}} = \frac{1}{16\pi \hat{s}^2} \cdot \left(\frac{G_F}{\sqrt{2}}\right)^2 \cdot L_{\mu\nu} H^{\mu\nu}}$$

WITH

NO POLARIZATION AVERAGE FOR ν_μ (LEFT-HANDED)

$$L_{\mu\nu} = \mathrm{Tr}\left\{ \gamma_\mu (1-\gamma_5) \not{k}' \gamma_\nu (1-\gamma_5) \not{k} \right\}$$

$$H^{\mu\nu} = \frac{1}{2} \mathrm{Tr}\left\{ \gamma^\mu (1-\gamma_5) \not{p}_q' \gamma^\nu (1-\gamma_5) \not{p}_q \right\}$$

AVERAGE OVER POLARIZATION OF INITIAL QUARK

- $$L_{\mu\nu} = 2\,\mathrm{Tr}\left\{ \gamma_\mu \not{k}' \gamma_\nu \not{k} \right\} + 2\,\mathrm{Tr}\left\{ \gamma_5 \gamma_\mu \not{k}' \gamma_\nu \not{k} \right\}$$

$$= 8 \left\{ k_\mu k'_\nu + k_\nu k'_\mu - k.k'\, g_{\mu\nu} + i\, \varepsilon_{\mu\nu\alpha\beta}\, k^\alpha k'^\beta \right\}$$

↳ SAME AS LEPTON TENSOR FOR e^- SCATTERING
WHEN PUTTING $(2h) = -1$ (ν ARE LEFT-HANDED)

- $$H^{\mu\nu} = \frac{8}{2} \left\{ P_q^\mu P_q'^\nu + P_q^\nu P_q'^\mu - P_q.P_q'\, g^{\mu\nu} + i\,\varepsilon^{\mu\nu\gamma\delta} (P_q)_\gamma (P_q')_\delta \right\}$$

- $$L_{\mu\nu} H^{\mu\nu} = 32 \left\{ k_\mu k'_\nu + k_\nu k'_\mu - \frac{Q^2}{2} g_{\mu\nu} \right\} \left\{ 2 P_q^\mu P_q^\nu - \frac{Q^2}{2} g^{\mu\nu} \right\}$$

$$- 32 \underbrace{\varepsilon_{\mu\nu\alpha\beta}\, \varepsilon^{\mu\nu\gamma\delta}}_{-2\left(g_\alpha^\gamma g_\beta^\delta - g_\alpha^\delta g_\beta^\gamma\right)}\, k^\alpha k'^\beta (P_q)_\gamma (P_q')_\delta$$

$$= 32 \left\{ 4 (P_q.k)(P_q.k') + \frac{1}{2}(Q^2)^2 \right.$$

$$\left. + 2 (P_q.k)(P_q'.k') - 2 (P_q.k')(P_q'.k) \right\}$$

$$= 32 \left\{ -\hat{s}\,\hat{u} + \frac{1}{2}(\hat{s}+\hat{u})^2 \right.$$

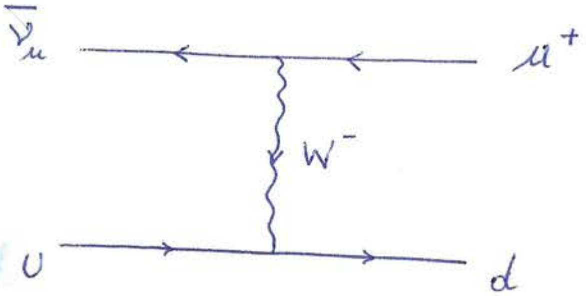
$$\left. + \frac{1}{2}\hat{s}^2 - \frac{1}{2}\hat{u}^2 \right\}$$

$$= 32\, \hat{s}^2$$

$$\hookrightarrow \boxed{\frac{d\sigma}{d\hat{t}}\left(\nu_\mu d \to \mu^- u\right) = \frac{G_F^2}{32\pi\hat{s}^2}\cdot 32\,\hat{s}^2 = \frac{G_F^2}{\pi}}$$

$$= \frac{d\sigma}{d\hat{t}}\left(\bar{\nu}_\mu \bar{d} \to \mu^+ \bar{u}\right)$$

$\hookrightarrow$ ANALOGOUSLY $\bar{\nu}_\mu u \to \mu^+ d$



ANTI-ν IS <u>RIGHT</u>-HANDED

W^- COUPLES ONLY TO <u>LEFT</u>-HANDED QUARKS

$$L_{\mu\nu} = \mathrm{Tr}\left\{\gamma_\nu(1-\gamma_5)\not{k}'\gamma_\mu(1-\gamma_5)\not{k}\right\}$$

$\hookrightarrow$ $\mu \leftrightarrow \nu$ COMPARED TO $L_{\mu\nu}$ FOR $\nu_\mu d \to \mu^- u$

$H^{\mu\nu}$ SAME AS FOR $\nu_\mu d \to \mu^- u$

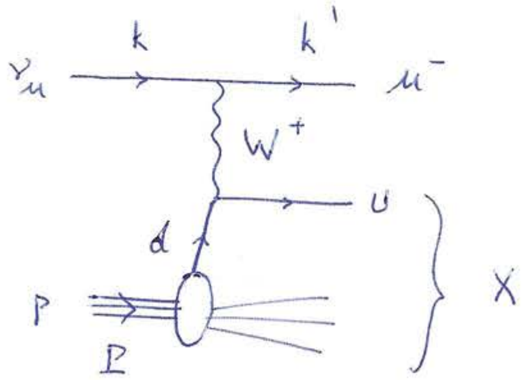
$$\therefore\; L_{\mu\nu}H^{\mu\nu} = 32\left\{-\hat{s}\hat{u} + \frac{1}{2}(\hat{s}+\hat{u})^2 - \left[\frac{1}{2}\hat{s}^2 - \frac{1}{2}\hat{u}^2\right]\right\} = 32\,\hat{u}^2$$

DIFFERENT RELATIVE SIGN

$$\boxed{\frac{d\sigma}{d\hat{t}}\left(\bar{\nu}_\mu u \to \mu^+ d\right) = \frac{G_F^2}{\pi}\cdot\frac{\hat{u}^2}{\hat{s}^2}}$$

$$= \frac{d\sigma}{d\hat{t}}\left(\nu_\mu \bar{u} \to \mu^- \bar{d}\right)$$

⇒ NEUTRINO – PROTON SCATTERING (NEGLECT ALL MASSES)



$$t = (k - k')^2$$
$$s = (k + P)^2$$

$$\frac{d\sigma}{dt}(\nu p \to \mu^- X) = \sum_q \int_0^1 dx \; q(x) \cdot \underbrace{\frac{d\sigma}{dt}(\nu q \to \mu^- q')}_{\text{WITH } \hat{s} = x s}$$

$\hat{s}$ ← PARTONIC

$$\boxed{\frac{d\sigma}{dt\, dx_B}(\nu p \to \mu^- X) = \sum_q q(x_B) \cdot \frac{d\sigma}{dt}(\nu q \to \mu^- q')\Big|_{\hat{s} = x_B s}}$$

↓ INTRODUCE DIMENSIONLESS VARIABLE

$$y \equiv \frac{q \cdot P}{k \cdot P} = \frac{2\, q \cdot P}{s} = \frac{Q^2}{s\, x_B}$$

$$\underline{\underline{Q^2 = s\, x_B\, y}} \qquad dQ^2 = s\, x_B\, dy$$

$$y = \frac{q \cdot (x P)}{k \cdot (x P)} = \frac{(k - k') \cdot P_q}{k \cdot P_q} = \frac{\hat{s} + \hat{u}}{\hat{s}}$$

↑ PARTONIC VARIABLES

$$\frac{\hat{u}}{\hat{s}} = -(1-y)$$

$$\Downarrow$$

$$\frac{d\sigma}{d\hat{t}}(\bar{\nu} u \to \mu^+ d) = \frac{G_F^2}{\pi}(1-y)^2$$

$$\frac{d\sigma}{dx_B\,dy}(\nu p \to \mu^- X)$$

$$= s\, x_B \cdot \frac{d\sigma}{dt\, dx_B}(\nu p \to \mu^- X)$$

$$= s\, x_B \cdot \frac{G_F^2}{\pi}\left\{ d(x_B) + \bar{u}(x_B) \cdot (1-y)^2 \right\}$$

$$\Longrightarrow \boxed{\begin{aligned} \frac{d\sigma}{dx_B\,dy}(\nu p \to \mu^- X) &= \frac{G_F^2 s}{\pi}\left\{ x_B\, d(x_B) + x_B\, \bar{u}(x_B)(1-y)^2 \right\} \\ \frac{d\sigma}{dx_B\,dy}(\bar{\nu} p \to \mu^+ X) &= \frac{G_F^2 s}{\pi}\left\{ x_B\, u(x_B)(1-y)^2 + x_B\, \bar{d}(x_B) \right\} \end{aligned}}$$

QUALITATIVE DIFFERENCE $\quad \nu$ CROSS SECTION $\sim d(x_B)$

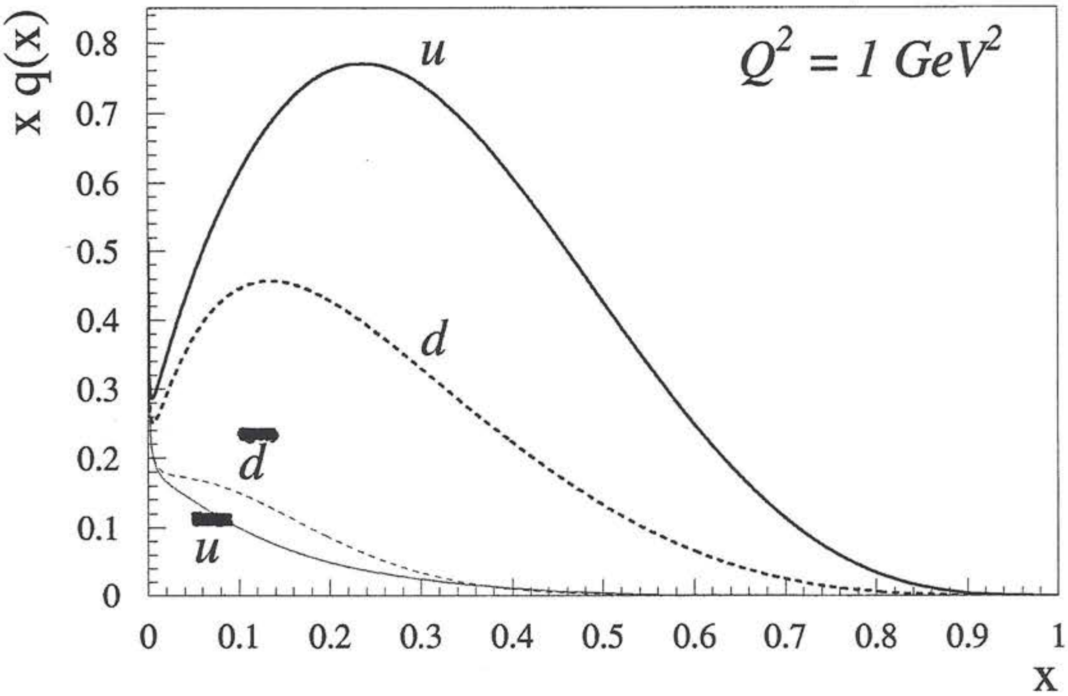
$\bar{\nu}$ CROSS SECTION $\sim \underline{\underline{u(x_B)(1-y)^2}}$

& VICE VERSA FOR $\bar{q}$

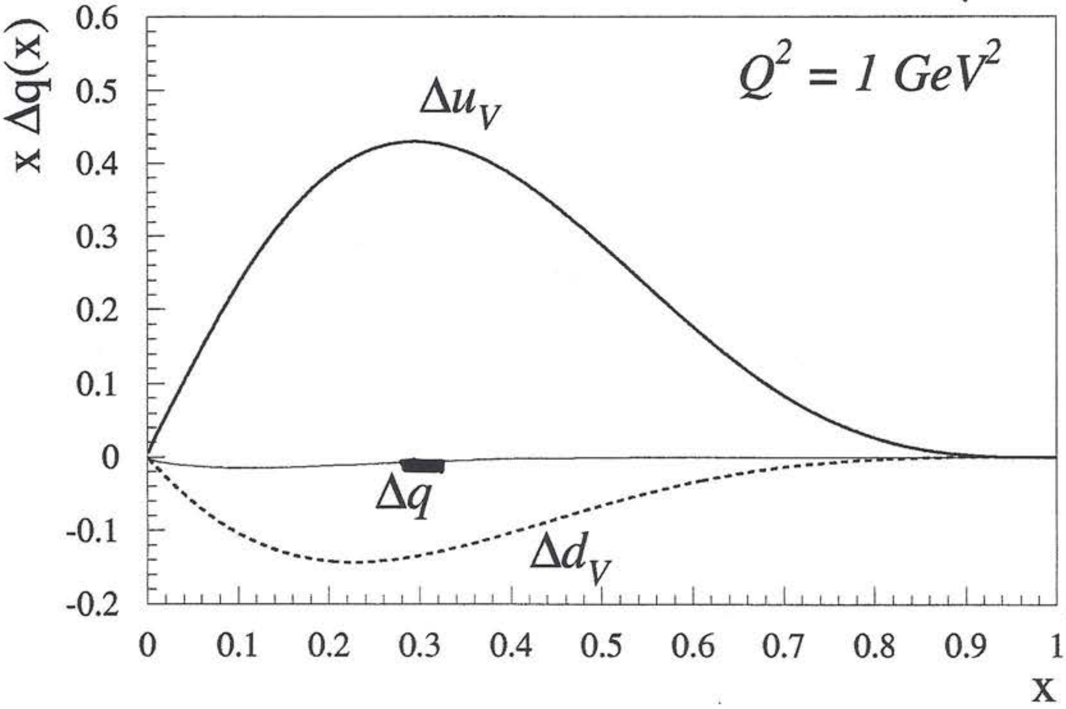
QUARK DISTRIBUTIONS in the PROTON

$q(x) = q_V(x) + q_S(x) \qquad q_S(x) = \bar{q}(x)$

⇒ MRST 98 NLO QUARK DISTR. $q(x)$



⇒ LEADER, SIDOROV, STAMENOV 98 NLO $\Delta q(x)$



UNPOLARIZED SUM RULES

$$q(x) = q_V(x) + q_S(x)$$
$$\bar{q}(x) = q_S(x)$$

⇒ **SUM RULES FOR 1 FLAVOR** (PROTON)

$$\int_0^1 dx\; u_V(x) = 2$$

$$\int_0^1 dx\; d_V(x) = 1$$

$$\int_0^1 dx\; [s(x) - \bar{s}(x)] = 0$$

$$|p\rangle = c_1 |uud\rangle + c_2 |uud u\bar{u}\rangle + c_3 |uud d\bar{d}\rangle + \ldots$$

⇒ **GROSS - LLEWELLYN - SMITH SUM RULE** (# VALENCE QUARK

$$S_{GLS} \equiv \frac{1}{2}\int_0^1 dx \left[F_3^{\nu p}(x,Q^2) + F_3^{\bar{\nu} p}(x,Q^2)\right]$$

$$= \int_0^1 dx \left[u(x,Q^2) - \bar{u}(x,Q^2) + d(x,Q^2) - \bar{d}(x,Q\right.$$

$$= 3\left[1 - \frac{\alpha_s(Q^2)}{\pi} + O(\alpha_s^2)\right]$$

(CCFR) EXP. $S_{GLS}(Q^2 = 3\,GeV^2) = 2.50 \pm 0.018\,(stat) \pm 0.078\,(s$

THEORY $S_{GLS}(Q^2 = 3\,GeV^2) = 2.66 \pm 0.04$ ✓

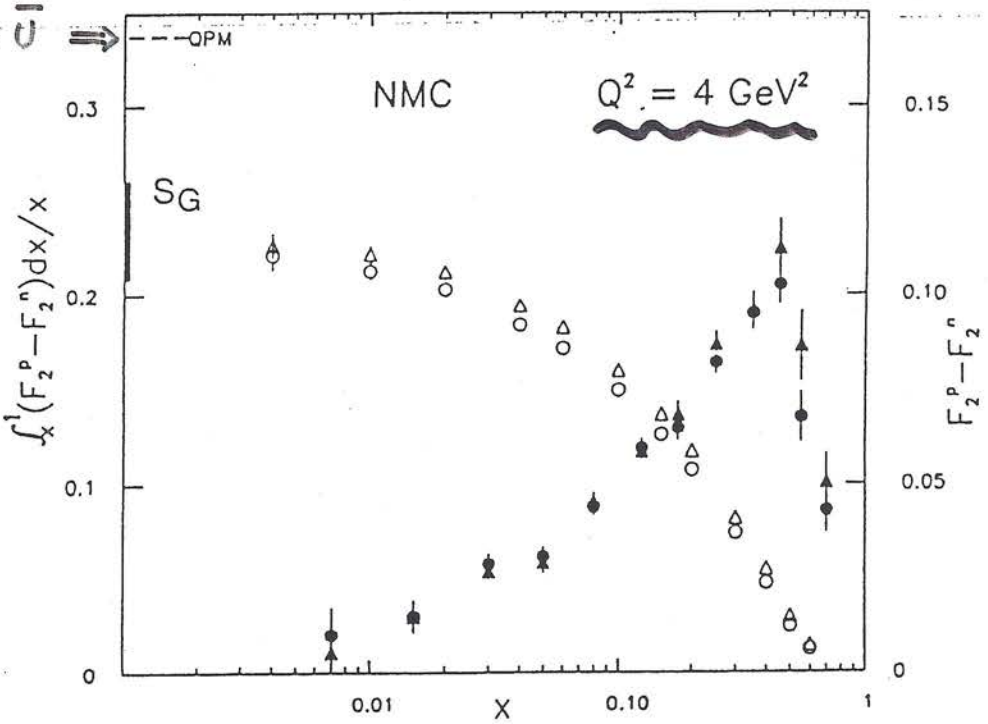
GOTTFRIED SUM RULE

$$\Rightarrow \quad F_2^p - F_2^n = \frac{x}{3}\left[u(x) + \bar{u}(x) - d(x) - \bar{d}(x)\right]$$

$$= \frac{x}{3}\left[u_v(x) - d_v(x)\right] + \frac{2x}{3}\left[\bar{u}(x) - \bar{d}(x)\right]$$

$$S_G \equiv \boxed{\int_0^1 dx\, \frac{1}{x}\left(F_2^p - F_2^n\right) = \frac{1}{3} - \frac{2}{3}\int_0^1 dx \left[\bar{d}(x) - \bar{u}(x)\right]}$$

IF $\bar{d} = \bar{u} \Longrightarrow$



(NMC 97) $\quad S_G = 0.2281 \pm 0.0065\ (stat)$

$$\Downarrow \qquad \bar{d} > \bar{u}$$

$$\int_0^1 dx\left(\bar{d} - \bar{u}\right) \simeq 0.15$$

MOMENTUM SUM RULE

$$\boxed{M_2^q(Q^2) \equiv \int_0^1 dx \; x \left[q(x,Q^2) + \bar{q}(x,Q^2) \right]}$$

→ MOMENTUM FRACTION OF PROTON CARRIED BY QUARK OF FLAVOR q

⇒ M_2^q AT LOW SCALE : $Q^2 = 1\ \mathrm{GeV}^2$

⇒ MRST 98 NLO QUARK DISTR.

q	$M_2^q\ (Q^2 = 1\ \mathrm{GeV}^2)$
u	0.40
d	0.22
s	0.03
SUM	0.65

⇒ M_2^q IN LIMIT $Q^2 \to \infty$

$$M_2^q(Q^2 \to \infty) = \frac{3N_f}{16 + 3N_f} \underset{N_f = 3}{=} 0.36$$

(FIXED POINT SOLUTION OF RENORMALIZATION GROUP EQ.)

∴ **GLUONS** CARRY AN IMPORTANT FRACTION OF PROTON MOMENTUM

MRST98 UNPOLARIZED parton distributions