STANDARD MODEL OF ELECTROWEAK INTERACTIONS

⇒ **GAUGE SYMMETRY GROUP U(1) OF ELECTROMAGNETIC INTERACTION**

ψ : DIRAC SPINOR FIELD

$$\mathcal{L}_0 = \bar{\psi} \left(i \gamma^\mu \partial_\mu - m \right) \psi$$

* REQUIRE THEORY TO BE INVARIANT UNDER LOCAL PHASE TRANSFORMATION $U(1)$

$$\boxed{\psi(x) \xrightarrow{U(1)} e^{ie\chi(x)} \cdot \psi(x)}$$

$\chi(x)$: SCALAR FUNCTION

$$\partial_\mu \psi \longrightarrow e^{ie\chi} \left\{ \partial_\mu \psi + ie(\partial_\mu \chi)\psi \right\}$$

$$\bar{\psi}\gamma^\mu(\partial_\mu \psi) \longrightarrow \bar{\psi}\gamma^\mu(\partial_\mu \psi) + \underbrace{ie\,\bar{\psi}\gamma^\mu(\partial_\mu \chi)\psi}_{\text{EXTRA TERM}}$$

$U(1)$ GAUGE INV. REQUIRES INTRODUCTION OF A VECTOR (GAUGE) FIELD THAT COMPENSATES EXTRA TERM

REPLACE: $\partial^\mu \Rightarrow \underset{\substack{\uparrow \\ \text{COVARIANT} \\ \text{DERIVATIVE}}}{D^\mu} = \partial^\mu + ie \underset{\substack{\uparrow \\ \text{GAUGE} \\ \text{FIELD}}}{A^\mu}$

$$A^\mu \xrightarrow{U(1)} A^\mu - \partial^\mu \chi$$

$$D^\mu \psi \xrightarrow{U(1)} e^{ie\chi} (D^\mu \psi)$$

$$\bar{\psi} \gamma_\mu (D^\mu \psi) \longrightarrow \bar{\psi} \gamma_\mu (D^\mu \psi) \qquad \text{INVARIANT}$$

$$\boxed{\mathcal{L}_0 + \mathcal{L}_{INT} = \bar{\psi} \left(i \gamma^\mu \partial_\mu - m - e \gamma^\mu A_\mu \right) \psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}}$$

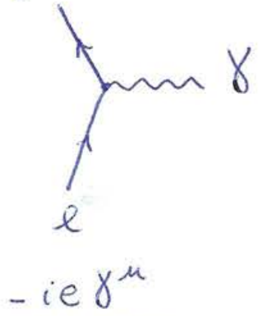
↑ U(1) INVARIANT

$$\underset{\substack{\uparrow \\ \text{FIELD TENSOR}}}{F^{\mu\nu}} = \partial^\mu A^\nu - \partial^\nu A^\mu$$

$$F^{\mu\nu} \xrightarrow{U(1)} F^{\mu\nu}$$

GAUGE FIELDS (γ)
MASSLESS

$$\mathcal{L}_{INT} = -e \underset{\substack{\uparrow \\ \text{E.M. CURRENT}}}{J^\mu_{e.m.}} A_\mu$$



$$J^\mu_{e.m.} = \bar{\psi} \gamma^\mu \psi$$

$-ie\gamma^\mu$

FEYNMAN RULE

⇒ GAUGE SYMMETRY GROUP $SU(2) \times U(1)$ OF ELECTROWEAK INTERACTIONS

- **WEAK ISOSPIN : $SU(2)$**

↳ TAKE $\psi_{\nu_e}^L$, ψ_ℓ^L TOGETHER (WEAK INTERACTIONS COUPLE BOTH)

WITH $\psi^L = \frac{1}{2}(1-\gamma_5)\psi$

$$\Psi_\ell^L = \begin{pmatrix} \psi_{\nu_e}^L \\ \psi_\ell^L \end{pmatrix}$$

↳ TAKE ψ_ℓ^R SEPARATE (NO ν_e^R IN SM)

$$\boxed{\mathcal{L}_0 = \bar{\Psi}_\ell^L \, i\gamma^\mu \partial_\mu \Psi_\ell^L + \bar{\psi}_\ell^R \, i\gamma^\mu \partial_\mu \psi_\ell^R}$$

$$= \bar{\psi}_{\nu_e}^L \, i\gamma^\mu \partial_\mu \psi_{\nu_e}^L + \bar{\psi}_\ell^L \, i\gamma^\mu \partial_\mu \psi_\ell^L + \bar{\psi}_\ell^R \, i\gamma^\mu \partial_\mu \psi_\ell^R$$

↳ GAUGE SYMMETRY: GAUGE GROUP $SU(2)$ WEAK ISOSPIN (t, t_3)

	t	t_3	
ν_e^L	$1/2$	$+1/2$	⇐ WEAK ISOSPIN DOUBLET
ℓ^L	$1/2$	$-1/2$	
ℓ^R	0	0	⇐ WEAK ISOSPIN SINGLET

REQUIRE THEORY TO BE INVARIANT UNDER

$$\boxed{\Psi_\ell^L \xrightarrow{SU(2)} \exp\{ i g \theta_i(x) t_i \} \Psi_\ell^L} \equiv U \Psi_\ell^L \qquad U \in SU(2)$$

$$\Downarrow \quad [t_i, t_j]_- = i \varepsilon_{ijk} t_k \qquad t_i = \frac{\tau_i}{2}$$

INVARIANCE OF $\mathcal{L}_0$ REQUIRES INTRODUCTION OF 3 VECTOR (GAUGE) FIELDS W_i^μ $(i = 1,2,3)$

$$\boxed{\partial^\mu \Psi_\ell^L \Longrightarrow \underbrace{\left(\partial^\mu + i g t_i W_i^\mu \right)}_{\equiv \mathcal{D}^\mu} \Psi_\ell^L}$$

SUCH THAT $\qquad \mathbb{W}^\mu \equiv t_i W_i^\mu$

$$\mathcal{D}^\mu \Psi_\ell^L \xrightarrow{SU(2)} \exp\{ i g \theta_i t_i \} \mathcal{D}^\mu \Psi_\ell^L \equiv U (\mathcal{D}^\mu \Psi_\ell^L)$$

$$\overline{\Psi}_\ell^L \gamma_\mu (\mathcal{D}^\mu \Psi_\ell^L) \xrightarrow{SU(2)} \overline{\Psi}_\ell^L \gamma_\mu (\mathcal{D}^\mu \Psi_\ell^L)$$

LEADS TO TF. PROPERTY OF GAUGE FIELDS

$$U (\mathcal{D}^\mu \Psi_\ell^L) = \partial^\mu (U \Psi_\ell^L) + i g \mathbb{W}'^\mu (U \Psi_\ell^L)$$

$$\parallel$$

$$U \left[\partial^\mu \Psi_\ell^L + i g \mathbb{W}^\mu \Psi_\ell^L \right]$$

$$U \left[\partial^\mu \Psi_\ell^L + i g \mathbb{W}^\mu \Psi_\ell^L \right]$$

$$= (\partial^\mu U) \Psi_\ell^L + U (\partial^\mu \Psi_\ell^L) + i g \mathbb{W}'^\mu (U \Psi_\ell^L)$$

$$\Downarrow$$

$$\mathbb{W}'^\mu U = U \mathbb{W}^\mu + \frac{i}{g}(\partial^\mu U)$$

$$\boxed{\mathbb{W}^\mu \xrightarrow[SU(2)]{} \mathbb{W}'^\mu = U \mathbb{W}^\mu U^{-1} + \frac{i}{g}(\partial^\mu U) U^{-1}}$$

$\downarrow$ INFINITESIMAL

$$U \simeq 1 + i g \theta_i t_i$$

$$t_i W_i'^\mu = t_i W_i^\mu + i g \underbrace{[t_j, t_k]_-}_{i \varepsilon_{ijk} t_i} \theta_j W_k^\mu + \frac{i}{g} i g (\partial^\mu \theta_i) t_i$$

$$\boxed{W_i'^\mu = W_i^\mu - (\partial^\mu \theta_i) - g \varepsilon_{ijk} \theta_j W_k^\mu}$$

AS IN QED

NEW TERM DUE TO NON-ABELIAN GAUGE GROUP SU(2)

$$\hookrightarrow \quad \mathcal{L}_0 + \mathcal{L}_{INT}$$

$$= \bar{\Psi}_\ell^L \, i \gamma^\mu D_\mu \Psi_\ell^L + \bar{\psi}_\ell^R \, i \gamma^\mu \partial_\mu \psi_\ell^R$$

$$= \mathcal{L}_0 - g \, \bar{\Psi}_\ell^L \gamma^\mu t_i \Psi_\ell^L \, W_{i\mu}$$

$$\mathcal{L}_{INT} = - g \Big\{ \bar{\Psi}_\ell^L \gamma^\mu t_1 \Psi_\ell^L \, W_{1\mu} + \bar{\Psi}_\ell^L \gamma^\mu t_2 \Psi_\ell^L \, W_{2\mu} + \bar{\Psi}_\ell^L \gamma^\mu t_3 \Psi_\ell^L \, W_{3\mu} \Big\}$$

INTRODUCE CHARGED GAUGE FIELDS

$$W^\mu \equiv \frac{1}{\sqrt{2}} \left(W_1^\mu - i W_2^\mu \right)$$ ANNIHILATES W^+, CREATES W^-

$$W^{\mu +} \equiv \frac{1}{\sqrt{2}} \left(W_1^\mu + i W_2^\mu \right)$$ ANNIHILATES W^-, CREATES W^+

$$\downarrow$$

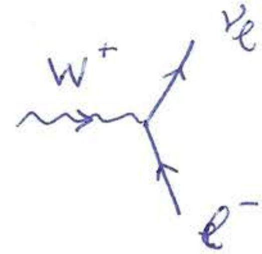
$$W_1^\mu = \frac{1}{\sqrt{2}} \left(W^\mu + W^{\mu +} \right)$$

$$W_2^\mu = \frac{i}{\sqrt{2}} \left(W^\mu - W^{\mu +} \right)$$

$$\mathcal{L}_{INT} = - g \Big\{ \bar{\Psi}_\ell^L \gamma^\mu (t_1 + i t_2) \Psi_\ell^L \, W_\mu \frac{1}{\sqrt{2}} + \bar{\Psi}_\ell^L \gamma^\mu (t_1 - i t_2) \Psi_\ell^L \, W_\mu^+ \frac{1}{\sqrt{2}} + \bar{\Psi}_\ell^L \gamma^\mu t_3 \Psi_\ell^L \, W_{3\mu} \Big\}$$

$$t_{\pm} = t_1 \pm i\, t_2$$

$$\bar{\Psi}_\ell^{\,L}\, \gamma^\mu\, t_+\, \Psi_\ell^{\,L} = \bar{\psi}_{\nu_\ell}^{\,L}\, \gamma^\mu\, \psi_\ell^{\,L}$$



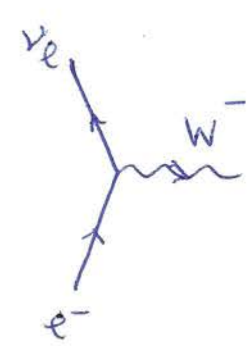
$$\bar{\Psi}_\ell^{\,L}\, \gamma^\mu\, t_-\, \Psi_\ell^{\,L} = \bar{\psi}_\ell^{\,L}\, \gamma^\mu\, \psi_{\nu_\ell}^{\,L}$$

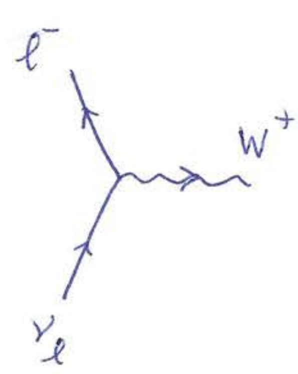
$$= \bar{\psi}_\ell\, \tfrac{1}{2}(1+\gamma_5)\, \gamma^\mu\, \tfrac{1}{2}(1-\gamma_5)\, \psi_{\nu_\ell}$$

$$= \tfrac{1}{2}\, \bar{\psi}_\ell\, \gamma^\mu (1-\gamma_5)\, \psi_{\nu_\ell}$$

$$\therefore\quad \boxed{\begin{aligned} \mathcal{L}_{INT}^{SU(2)} = &-\frac{g}{2\sqrt{2}}\, \bar{\psi}_{\nu_\ell}\, \gamma^\mu (1-\gamma_5)\, \psi_\ell\, W_\mu \\ &-\frac{g}{2\sqrt{2}}\, \bar{\psi}_\ell\, \gamma^\mu (1-\gamma_5)\, \psi_{\nu_\ell}\, W_\mu^+ \\ &-\frac{g}{2(2)}\left\{ \bar{\psi}_{\nu_\ell}\, \gamma^\mu (1-\gamma_5)\, \psi_{\nu_\ell} - \bar{\psi}_\ell\, \gamma^\mu (1-\gamma_5)\, \psi_\ell \right\} W_{3\mu} \end{aligned}}$$

FIRST 2 TERMS DESCRIBE CHARGED WEAK INTERACTION





FEYNMAN RULE

$$\boxed{-i\, \frac{g}{2\sqrt{2}}\, \gamma^\mu (1-\gamma_5)}$$

• WEAK HYPERCHARGE : $U(1)_Y$

2 NEUTRAL CURRENTS

$\rightarrow$ $J_3^\mu = \bar{\Psi}_\ell^L \gamma^\mu t_3 \Psi_\ell^L$ $\rightsquigarrow$ COUPLES TO W_3

$$= \frac{1}{2} \bar{\psi}_{\nu_\ell}^L \gamma^\mu \psi_{\nu_\ell}^L - \frac{1}{2} \bar{\psi}_\ell^L \gamma^\mu \psi_\ell^L$$

$\rightarrow$ $J_{em}^\mu = \bar{\psi}_\ell^L \gamma^\mu Q \psi_\ell^L + \bar{\psi}_\ell^R \gamma^\mu Q \psi_\ell^R$ $\rightsquigarrow$ COUPLES TO γ

INTRODUCE 'WEAK HYPERCHARGE' Y SUCH THAT

$$Q = t_3 + \frac{1}{2} Y$$

Q ↑ ELECTRIC CHARGE; t_3 ↑ WEAK ISOSPIN; Y ↑ WEAK HYPERCHARGE

	t	t_3	Y	Q
ν_ℓ^L	1/2	+1/2	−1	0
ℓ^L	1/2	−1/2	−1	−1
ℓ^R	0	0	−2	−1

$$\frac{1}{2} J_Y^\mu = J_{em}^\mu - J_3^\mu$$

↳ INVARIANCE OF $\mathcal{L}$ UNDER $U(1)_Y$

$$\boxed{\psi_e \xrightarrow{U(1)_Y} \exp\left\{ i g' \chi(x) \frac{Y}{2} \right\} \psi_e}$$

$\Downarrow$

$$\boxed{\partial^\mu \psi_e \Rightarrow \left(\partial^\mu + i g' \frac{Y}{2} B^\mu \right) \psi_e}$$

GAUGE FIELD

$$B^\mu \xrightarrow{U(1)} B^\mu - \partial^\mu \chi$$

$$\boxed{\mathcal{L}_{INT}^{U(1)_Y} = - g' \left\{ \bar{\psi}_{\nu_e}^L \gamma^\mu \left(\frac{-1}{2}\right) \psi_{\nu_e}^L + \bar{\psi}_e^L \gamma^\mu \left(-\frac{1}{2}\right) \psi_e^L + \bar{\psi}_e^R \gamma^\mu \left(-\frac{2}{2}\right) \psi_e^R \right\} B_\mu}$$

NEUTRAL CURRENT

$$\mathcal{L}_{INT}^{NC} = - g \bar{\Psi}_e^L \gamma^\mu t_3 \Psi_e^L \; W_{3\mu} - g' \bar{\Psi}_e^L \gamma^\mu \left(\frac{Y}{2}\right) \Psi_e^L B_\mu - g' \bar{\psi}_e^R \gamma^\mu \left(\frac{Y}{2}\right) \psi_e^R B_\mu$$

$\downarrow$

REPLACE $\frac{1}{2} Y = Q - t_3$

$$\mathcal{L}_{INT}^{NC} = - \bar{\Psi}_e^L \gamma^\mu t_3 \Psi_e^L \left(g W_{3\mu} - g' B_\mu \right)$$

$$- g' B_\mu \underbrace{\left(\bar{\psi}_e^L \gamma^\mu Q \psi_e^L + \bar{\psi}_e^R \gamma^\mu Q \psi_e^R \right)}_{\bar{\psi}_e \gamma^\mu Q \psi_e \equiv J_{em}^\mu}$$

$\uparrow$ E.M. CURRENT

$$\boxed{\mathcal{L}_{INT}^{NC} = - \bar{\Psi}_e^L \gamma^\mu t_3 \Psi_e^L \left(g W_{3\mu} - g' B_\mu \right) - J_{em}^\mu g' B_\mu}$$

PHYSICAL GAUGE BOSON FIELDS (γ, Z^0)
ARE LINEAR COMBINATIONS OF (W_3, B)
SUCH THAT $g W_{3\mu} - g' B_\mu$ DOES NOT CONTAIN γ
(BECAUSE IT COUPLES TO NEUTRINO)

$\to$ ELECTROWEAK UNIFICATION

- **ELECTROWEAK UNIFICATION** : GLASHOW-WEINBERG-SALAM MODEL (1967, 1968)

↳ PHYSICAL GAUGE BOSON FIELDS ARE MIXTURE OF W_3, B

$\Downarrow$

MIXING ANGLE : **WEINBERG ANGLE θ_W**

→ PHYSICAL GAUGE BOSON FIELDS

$$\boxed{\begin{pmatrix} W_3^\mu \\ B^\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} Z^\mu \\ A^\mu \end{pmatrix}}$$

↳ $g W_{3\mu} - g' B_\mu$ SHOULD NOT CONTAIN A_μ

$$= \left(g\cos\theta_W + g'\sin\theta_W\right) Z_\mu + \underbrace{\left(g\sin\theta_W - g'\cos\theta_W\right)}_{=0} A_\mu$$

$$\boxed{g\sin\theta_W = g'\cos\theta_W} \qquad \tan\theta_W = \frac{g'}{g}$$

↳ $g' B_\mu$

$$= -g'\sin\theta_W \, Z_\mu + g'\cos\theta_W \, A_\mu$$

$g' B_\mu$ COUPLES TO E.M. CURRENT $\Rightarrow$ $\boxed{g'\cos\theta_W = e}$

↳ $\mathcal{L}_{INT}^{NC}$ IN TERMS OF A^μ & Z^μ

$$\mathcal{L}_{INT}^{NC} = -\bar{\Psi}_\ell^L \gamma^\mu t_3 \Psi_\ell^L \left(\frac{g}{\cos\theta_W}\right) Z_\mu$$

$$- J_{em}^\mu \left(-\frac{g}{\cos\theta_W}\sin^2\theta_W\right) Z_\mu - e J_{em}^\mu A_\mu$$

⇓

$$\boxed{\mathcal{L}_{INT}^{NC} = -e J_{em}^\mu A_\mu - \frac{g}{\cos\theta_W}\underbrace{\left(J_3^\mu - \sin^2\theta_W J_{em}^\mu\right)}_{J_{NC}^\mu} Z_\mu}$$

⇑

NEUTRAL WEAK CURRENT

1° TERM : E.M. INTERACTION

2° TERM : NEUTRAL WEAK INTERACTION

↓

NEUTRAL WEAK CURRENT J_{NC}^μ

$$J_{NC}^\mu = \frac{1}{4}\bar{\psi}_{\nu_e}\gamma^\mu(1-\gamma_5)\psi_{\nu_e}$$

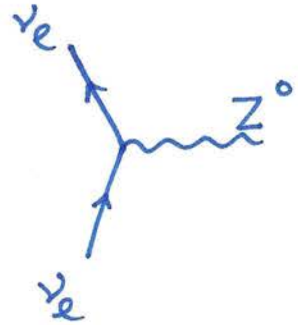
$$-\frac{1}{4}\bar{\psi}_e\gamma^\mu(1-\gamma_5)\psi_e + \sin^2\theta_W\bar{\psi}_e\gamma^\mu\psi_e.$$

↓

(LEPTON HAS NEGATIVE CHARGE)

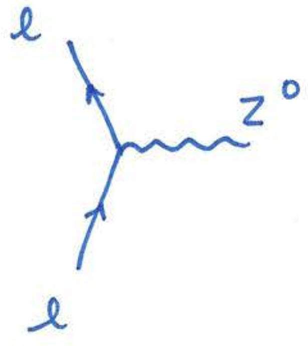
$$\boxed{\begin{aligned} J^{\mu}_{NC} = & \ \frac{1}{4}\,\bar{\psi}_{\nu_\ell}\,\gamma^{\mu}(1-\gamma_5)\,\psi_{\nu_\ell} \\ & - \frac{1}{4}\,\bar{\psi}_{\ell}\,\gamma^{\mu}\left(\left[1-4\sin^2\theta_W\right]-\gamma_5\right)\psi_{\ell} \end{aligned}}$$

↳ FEYNMAN RULES : NEUTRAL WEAK INTERACTION



$$\boxed{-\,i\,\frac{g}{4\cos\theta_W}\,\gamma^{\mu}(1-\gamma_5)}$$

(V − A)



$$\boxed{+\,i\,\frac{g}{4\cos\theta_W}\,\gamma^{\mu}\left(\underbrace{\left[1-4\sin^2\theta_W\right]}_{c_V}-\gamma_5\right)}$$

ELECTROWEAK INTERACTIONS

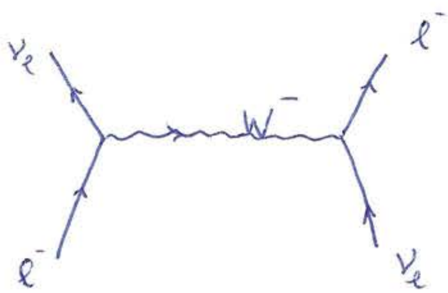
DESCRIBED BY 1 COUPLING : e

& 1 MIXING ANGLE $\sin\theta_W$ ⇒ TO BE DETERMINED BY EXPERIMENT

g & g' FOLLOW FROM $\boxed{g\sin\theta_W = g'\cos\theta_W = e}$

⇒ GAUGE BOSON MASSES

•



$$\frac{G_F}{\sqrt{2}} = \left(\frac{g}{2\sqrt{2}}\right)^2 \cdot \frac{1}{M_W^2} = \frac{g^2}{8 M_W^2}$$

$$\downarrow \qquad g = \frac{e}{\sin \Theta_W}$$

$$\frac{G_F}{\sqrt{2}} = \frac{e^2}{8 \sin^2 \Theta_W} \quad \frac{1}{M_W^2}$$

$\downarrow$ G_F KNOWN FROM n β- DECAY

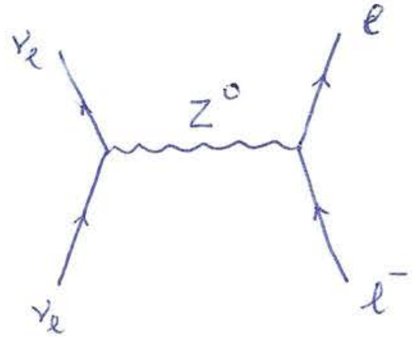
$$\boxed{M_{W^\pm}^2 = \frac{e^2 \sqrt{2}}{8\, G_F \sin^2 \Theta_W}}$$

PREDICTED ONCE $\sin^2 \Theta_W$ IS KNOWN !

PRESENT DAY VALUE $\underline{\underline{M_{W^\pm} \simeq 80.423 \pm 0.039 \text{ GeV}}}$

OBSERVED CERN (1983)

PDG 2008 $M_W = 80.398 \pm 0.025$ GeV

- 

$$\begin{pmatrix} Z^\mu \\ A^\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} W_3^\mu \\ B^\mu \end{pmatrix}$$

$$\begin{cases} Z^\mu = \cos\theta_W \; W_3^\mu - \sin\theta_W \; B^\mu \\ A^\mu = \sin\theta_W \; W_3^\mu + \cos\theta_W \; B^\mu \end{cases}$$

BOSONS : FIELD EQ. INVOLVE SQUARE OF MASSES

INTERFERENCE

$$M_Z^2 = \cos^2\theta_W \; M_W^2 + \sin^2\theta_W \; M_B^2 - 2\sin\theta_W\cos\theta_W \; M_{BW}^2$$

$$0 = M_\gamma^2 = \sin^2\theta_W \; M_W^2 + \cos^2\theta_W \; M_B^2 + 2\sin\theta_W\cos\theta_W \; M_{BW}^2$$

BOTH STATES ARE ORTHOGONAL $\longrightarrow$ $0 = \sin\theta_W\cos\theta_W\,(M_W^2 - M_B^2) + M_{BW}^2\,(\cos^2\theta_W - \sin^2\theta_W)$

$\downarrow$ ELIMINATE M_{BW}^2

- $$M_{BW}^2 = (M_B^2 - M_W^2)\,\frac{\sin\theta_W \cos\theta_W}{\cos^2\theta_W - \sin^2\theta_W}$$

- $(1) + (2)$ $\quad M_Z^2 = M_W^2 + M_B^2$

$$\cos^2\theta_W \; M_Z^2 = (\cos^2\theta_W - \sin^2\theta_W)\, M_W^2$$

$$- \frac{2\sin^2\theta_W \cos^2\theta_W}{(\cos^2\theta_W - \sin^2\theta_W)} \cdot (M_Z^2 - 2M_W^2)$$

$$\downarrow$$

$$\underbrace{(\cos^4\theta_W + \sin^2\theta_W \cos^2\theta_W)}_{\cos^2\theta_W} M_Z^2 = (\cos^4\theta_W + \sin^4\theta_W + 2\sin^2\theta_W\cos^2\theta_W)\, M_W^2$$

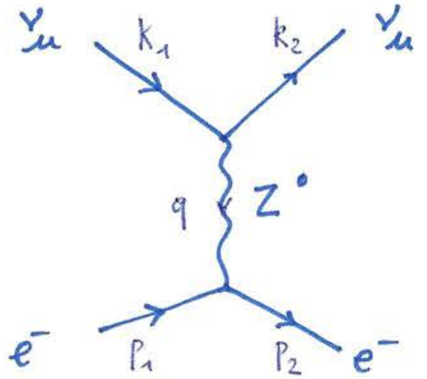
$$\boxed{M_Z^2 = \frac{M_W^2}{\cos^2\theta_W}}$$

$$M_{Z^0} \simeq 91.1876 \pm 0.0021 \text{ GeV}$$

⇒ DETERMINATION OF $\sin^2\Theta_W$

• $\nu_\mu e^- \to \nu_\mu e^-$ ELASTIC SCATTERING

↳ CAN ONLY PROCEED VIA NEUTRAL CURRENT WEAK INTERACTION



$$\longrightarrow \quad -i\frac{g}{4\cos\Theta_W}\gamma^\mu(1-\gamma_5)$$

$$\longrightarrow \quad i\frac{g}{4\cos\Theta_W}\gamma^\mu(c_V-\gamma_5)$$

$$c_V \equiv 1-4\sin^2\Theta_W$$

$$s \equiv (k_1+p_1)^2$$
$$t \equiv (k_1-k_2)^2 \ll M_Z^2$$

+ NEGLECT e^- MASS

$$\Rightarrow \mathcal{M} \simeq \frac{ig^2}{16\cos^2\Theta_W}\frac{1}{M_Z^2}\cdot \bar{u}(k_2,s_\nu')\gamma^\mu(1-\gamma_5)u(k_1,s_\nu)$$
$$\cdot\,\bar{u}(p_2,s_e')\gamma_\mu(c_V-\gamma_5)u(p_1,s_e)$$

$$\downarrow \quad M_Z^2\cos^2\Theta_W = M_W^2$$

$$\frac{g^2}{8M_W^2} = \frac{G_F}{\sqrt{2}}$$

$$\boxed{\mathcal{M} = i\frac{G_F}{2\sqrt{2}}\cdot\bar{u}(k_2,s_\nu')\gamma^\mu(1-\gamma_5)u(k_1,s_\nu)\cdot\bar{u}(p_2,s_e')\gamma_\mu(c_V-\gamma_5)u(p_1,s_e)}$$

$$\Rightarrow \frac{1}{2} \sum_{s_\nu s_e} \sum_{s'_\nu s'_e} |\mathcal{M}|^2$$

ONLY 1 ν SPIN STATE

$$= \frac{G_F^2}{4} \cdot \frac{1}{4} \, \mathrm{Tr}\left\{ \gamma^\mu (1-\gamma_5) \not{k}_1 \, \gamma^\nu (1-\gamma_5) \not{k}_2 \right\}$$

$$\cdot \, \mathrm{Tr}\left\{ \gamma_\mu (c_V - \gamma_5) \not{p}_1 \, \gamma_\nu (c_V - \gamma_5) \not{p}_2 \right\}$$

$$= \frac{G_F^2}{4} \, \mathrm{Tr}\left\{ (1+\gamma_5) \gamma^\mu \not{k}_1 \gamma^\nu \not{k}_2 \right\}$$

$$\cdot \, \mathrm{Tr}\left\{ \left[\frac{1}{2}(c_V^2+1) + \gamma_5 c_V \right] \gamma_\mu \not{p}_1 \gamma_\nu \not{p}_2 \right\}$$

$$= 4 G_F^2 \left\{ k_1^\mu k_2^\nu + k_1^\nu k_2^\mu - (k_1 \cdot k_2) g^{\mu\nu} + i \varepsilon^{\mu\alpha\nu\beta} k_{1\alpha} k_{2\beta} \right\}$$

$$\cdot \left\{ \frac{1}{2}(1+c_V^2) \left[P_{1\mu} P_{2\nu} + P_{1\nu} P_{2\mu} - P_1 \cdot P_2 \, g_{\mu\nu} \right] \right.$$

$$\left. + \; c_V \, i \, \varepsilon_{\mu\gamma\nu\delta} P_1^\gamma P_2^\delta \right\}$$

$$= 4 G_F^2 \left\{ \frac{1}{2}(1+c_V^2) 2 \left[(k_1 \cdot P_1)(k_2 \cdot P_2) + (k_1 \cdot P_2)(k_2 \cdot P_1) \right] \right.$$

$$\left. + \; c_V \, 2 \left[(k_1 \cdot P_1)(k_2 \cdot P_2) - (k_1 \cdot P_2)(k_2 \cdot P_1) \right] \right\}$$

$$s = 2 k_1 \cdot P_1 = 2 k_2 \cdot P_2 .$$

$$\frac{P_1 \cdot k_2}{P_1 \cdot k_1} = \frac{P_2 \cdot k_1}{P_2 \cdot k_2} \equiv 1 - Y$$

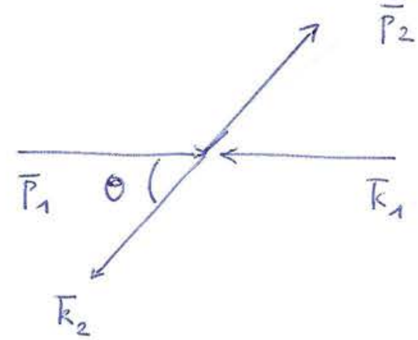
$\hookrightarrow$ DIMENSIONLESS

$$= 2 G_F^2 s^2 \left\{ \frac{1}{2} (1 + c_V^2) \left[1 + (1-y)^2 \right] \right.$$

$$\left. + \quad c_V \quad \left[1 - (1-y)^2 \right] \right\}$$

$$= G_F^2 s^2 \left\{ (1 + c_V)^2 + (1 - c_V)^2 (1-y)^2 \right\}$$

$\Rightarrow$ C.M.



$$|\bar{p}_1| = |\bar{p}_2| = |\bar{k}_1| = |\bar{k}_2| = \frac{\sqrt{s}}{2}$$

$$\frac{p_1 \cdot k_2}{p_1 \cdot k_1} = \frac{1}{2} (1 + \cos\theta) \quad \Rightarrow \quad \frac{1}{2} d\cos\theta = dy$$

$$y = \sin^2 \frac{\theta}{2}$$

$$d\sigma = \frac{1}{2|\bar{p}_1|\, 2|\bar{k}_1| \cdot 2} \cdot \frac{d^3 \bar{k}_2}{(2\pi)^3 2 |\bar{k}_2|} \cdot \frac{d^3 \bar{p}_2}{(2\pi)^3 2 |\bar{p}_2|} (2\pi)^4 \delta^4(p_1 + k_1 - p_2 - k_2) \cdot \frac{1}{2} \sum |\mathcal{M}|^2$$

$$= \frac{1}{2s} \frac{1}{(2\pi)^2} \frac{d^3 \bar{k}_2}{s} \underbrace{\delta(\sqrt{s} - 2|\bar{k}_2|)}_{\frac{1}{2}\delta\left(|\bar{k}_2| - \frac{\sqrt{s}}{2}\right)} \cdot \frac{1}{2} \sum |\mathcal{M}|^2$$

$$d\sigma = \frac{1}{2s}\frac{1}{(2\pi)^2}\frac{1}{8}\, d\cos\theta\, d\phi$$

$$\cdot\ G_F^2 s^2 \left\{ (1+c_V)^2 + (1-c_V)^2(1-y)^2 \right\}$$

$$\Downarrow$$

$e^- \nu_\mu \to e^- \nu_\mu$

$$\boxed{\frac{d\sigma}{dy} = \frac{G_F^2 s}{16\pi}\left\{ (1+c_V)^2 + (1-c_V)^2(1-y)^2 \right\}}$$

FOR $e^- \bar{\nu}_\mu \to e^- \bar{\nu}_\mu$: $c_V \to -c_V$

$$\Rightarrow \quad \underline{\underline{\sin^2\theta_W \simeq 0.23113 \pm 6.5\times 10^{-5}}}$$

⇒ QUARK ELECTROWEAK CURRENTS : $SU(2) \times U(1)$

• MULTIPLETS FOR $SU(2) \times U(1)$

↳ $\Psi^L_{q_1} = \begin{pmatrix} u^L \\ d^L_c \end{pmatrix}$ WEAK ISOSPIN DOUBLET

↑ 1° FAMILY

$\Psi^L_{q_2} = \begin{pmatrix} c^L \\ s^L_c \end{pmatrix}$

↑ 2° FAMILY

WITH $\begin{pmatrix} d^L_c \\ s^L_c \end{pmatrix} = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} d^L \\ s^L \end{pmatrix}$

↑ WEAK INTERACTION EIGENSTATES

↑ STRONG INTERACTION (MASS) EIGENSTATES

→ θ_c : CABIBBO ANGLE $\cos\theta_c \simeq 0.975$

$\sin\theta_c \simeq 0.221$

→ FOR 3 FAMILIES OF QUARKS : CKM MATRIX

↳ RIGHT HANDED QUARKS ARE SINGLETS

u^R

d^R

- QUANTUM NUMBERS

$$Q = t_3 + \frac{1}{2} Y \qquad \left(\frac{1}{2} Y \equiv Q - t_3 \right)$$

$\uparrow$ ELECTRIC CHARGE $\quad \uparrow$ WEAK ISOSPIN $\quad \uparrow$ WEAK HYPERCHARGE

	t	t_3	Y	Q
u^L	1/2	+1/2	+1/3	+2/3
d^L_c	1/2	−1/2	+1/3	−1/3
u^R	0	0	+4/3	+2/3
d^R	0	0	−2/3	−1/3

- WEAK ISOSPIN SU(2)

$$\mathcal{L}_0 = \bar{\Psi}^L_q \, i\gamma^\mu \partial_\mu \Psi^L_q + \bar{u}^R i\gamma^\mu \partial_\mu u^R + \bar{d}^R i \gamma^\mu \partial_\mu d^R$$

MINIMAL SUBSTITUTION

$$\Downarrow \qquad \partial_\mu \Rightarrow D_\mu \equiv \partial_\mu + i g \, t_i W_{i\mu}$$

$$\mathcal{L}^{SU(2)}_{INT} = - g \, \bar{\Psi}^L_q \gamma^\mu t_i \Psi^L_q W_{i\mu}$$

INTRODUCE W^μ & $W^{\mu +}$ FIELDS

THAT ANNIHILATE / CREATE $W^\pm$

$$W^\mu \equiv \frac{1}{\sqrt{2}} \left(W_1^\mu - i W_2^\mu \right)$$

$$W^{\mu +} \equiv \frac{1}{\sqrt{2}} \left(W_1^\mu + i W_2^\mu \right)$$

$$\downarrow$$

$$W_1^\mu = \frac{1}{\sqrt{2}} \left(W^\mu + W^{\mu +} \right)$$

$$W_2^\mu = \frac{i}{\sqrt{2}} \left(W^\mu - W^{\mu +} \right)$$

$$\hookrightarrow \mathcal{L}_{INT}^{SU(2)} = - \frac{g}{\sqrt{2}} \bar{\Psi}_q^L \gamma^\mu \left(t_1 + i t_2 \right) \Psi_q^L W_\mu - \frac{g}{\sqrt{2}} \bar{\Psi}_q^L \gamma^\mu \left(t_1 - i t_2 \right) \Psi_q^L W_\mu^+ - g \bar{\Psi}_q^L \gamma^\mu t_3 \Psi_q^L W_{3\mu}$$

$$\boxed{\mathcal{L}_{INT}^{SU(2)} = - \frac{g}{\sqrt{2}} \bar{u}^L \gamma^\mu d_c^L W_\mu - \frac{g}{\sqrt{2}} \bar{d}_c^L \gamma^\mu u^L W_\mu^+ - g \bar{\Psi}_q^L \gamma^\mu t_3 \Psi_q^L W_{3\mu}}$$

FIRST 2 TERMS ARE CHARGED CURRENT WEAK INTERACTION

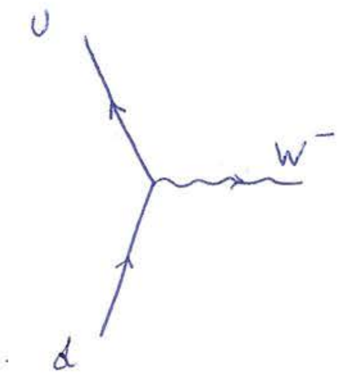
NOTE: $\bar{u}^L \gamma^\mu d_c^L = \frac{1}{2} \bar{u} \underbrace{\gamma^\mu (1 - \gamma_5)}_{V-A} d_c$

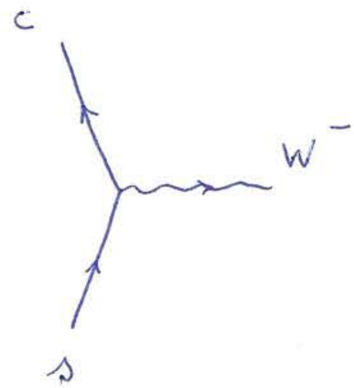
- FEYNMAN RULES :

CHARGED CURRENT WEAK INTERACTION

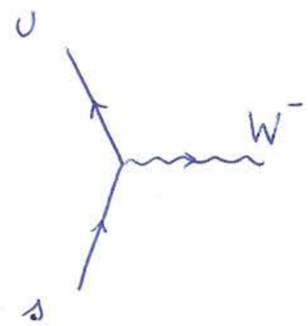
$$d_c = \cos\theta_c \; d + \sin\theta_c \; s$$

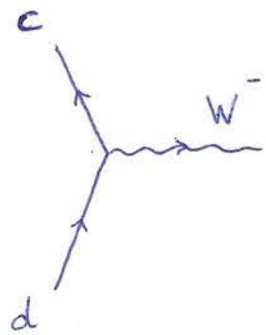
$$s_c = -\sin\theta_c \; d + \cos\theta_c \; s$$





$$\boxed{-\, i\,\frac{g}{2\sqrt{2}} \cdot \cos\theta_c \cdot \gamma^\mu (1-\gamma_5)}$$





$$\boxed{-\, i\,\frac{g}{2\sqrt{2}} \left(\pm \sin\theta_c\right) \cdot \gamma^\mu (1-\gamma_5)}$$

$(+\sin\theta_c)$ $(-\sin\theta_c)$

∴ CC INTERACTION IS

1/ OF TYPE V − A

2/ FLAVOR CHANGING

- **WEAK HYPERCHARGE $U(1)$**

$$\mathcal{L}_0 = \bar{\Psi}_q^L \, i\gamma^\mu \partial_\mu \Psi_q^L + \bar{u}^R i\gamma^\mu \partial_\mu u^R + \bar{d}^R i\gamma^\mu \partial_\mu d^R$$

$$\Downarrow \qquad \partial_\mu \Rightarrow D_\mu = \partial_\mu + i g \underbrace{t_i W_{i\mu}}_{SU(2)} + i g' \underbrace{\frac{Y}{2} B_\mu}_{U(1)}$$

$$\mathcal{L}_{INT}^{U(1)} = \Big\{ - g' \bar{\Psi}_q^L \gamma^\mu \underbrace{\frac{Y}{2}}_{\frac{1}{6}} \Psi_q^L - g' \bar{u}^R \gamma^\mu \underbrace{\frac{Y}{2}}_{+\frac{2}{3}} u^R - g' \bar{d}^R \gamma^\mu \underbrace{\frac{Y}{2}}_{-\frac{1}{3}} d^R \Big\} \times B_\mu$$

NEUTRAL CURRENT

$$\mathcal{L}_{INT}^{NC} = - g \bar{\Psi}_q^L \gamma^\mu t_3 \Psi_q^L W_{3\mu}$$
$$- g' \bar{\Psi}_q^L \gamma^\mu \frac{Y}{2} \Psi_q^L B_\mu - g' \bar{u}^R \gamma^\mu \frac{Y}{2} u^R B_\mu - g' \bar{d}^R \gamma^\mu \frac{Y}{2} d^R B_\mu$$

$\Downarrow$ REPLACE $\frac{Y}{2} = Q - t_3$

$$\mathcal{L}_{INT}^{NC} = - \bar{\Psi}_q^L \gamma^\mu t_3 \Psi_q^L \left(g W_{3\mu} - g' B_\mu \right)$$
$$- g' B_\mu \underbrace{\Big(\bar{u}^L \gamma^\mu Q u^L + \bar{u}^R \gamma^\mu Q u^R + \bar{d}^L \gamma^\mu Q d^L + \bar{d}^R \gamma^\mu Q d^R \Big)}_{J^\mu_{em} = \bar{u} \gamma^\mu Q u + \bar{d} \gamma^\mu Q d}$$

$$J^\mu_{em} = \bar{u} \gamma^\mu \underset{+2/3}{\underset{\shortparallel}{Q}} u + \bar{d} \gamma^\mu \underset{-1/3}{\underset{\shortparallel}{Q}} d$$

$$\mathcal{L}_{INT}^{NC} = - \bar{\Psi}_q^L \gamma^\mu t_3 \Psi_q^L \left(g W_{3\mu} - g' B_\mu \right) - g' B_\mu J^\mu_{em}$$

INTRODUCE Z^0, γ FIELDS

$$\begin{pmatrix} W_3^\mu \\ B^\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} Z^\mu \\ A^\mu \end{pmatrix}$$

$$g \sin\theta_W = g' \cos\theta_W = e$$

$$\hookrightarrow \quad g W_{3\mu} - g' B_\mu = \left(g \cos\theta_W + g' \sin\theta_W \right) Z_\mu = \frac{g}{\cos\theta_W} Z_\mu$$

$$\hookrightarrow \quad g' B_\mu = - g' \sin\theta_W Z_\mu + g' \cos\theta_W A_\mu = - \frac{g}{\cos\theta_W} \sin^2\theta_W Z_\mu + e A_\mu$$

$$\therefore \quad \boxed{\mathcal{L}_{INT}^{NC} = - e A_\mu J^\mu_{em} - \frac{g}{\cos\theta_W} \left(J_3^\mu - \sin^2\theta_W J^\mu_{em} \right) Z_\mu}$$

WITH

$$J^\mu_{em} = +\frac{2}{3}\,\bar{u}\gamma^\mu u - \frac{1}{3}\,\bar{d}\gamma^\mu d$$

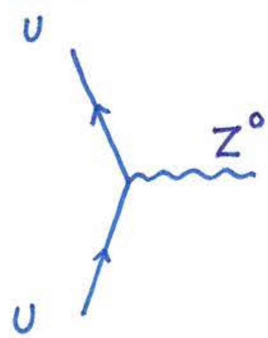
$$J^\mu_3 = \frac{1}{4}\,\bar{u}\gamma^\mu(1-\gamma_5)u - \frac{1}{4}\,\bar{d}\gamma^\mu(1-\gamma_5)d.$$

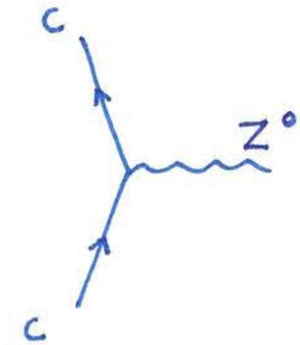
$$J^\mu_3 - \sin^2\theta_W\, J^\mu_{em}$$

$$= \frac{1}{4}\,\bar{u}\gamma^\mu\left(\left[1-\frac{8}{3}\sin^2\theta_W\right]-\gamma_5\right)u$$

$$-\frac{1}{4}\,\bar{d}\gamma^\mu\left(\left[1-\frac{4}{3}\sin^2\theta_W\right]-\gamma_5\right)d$$

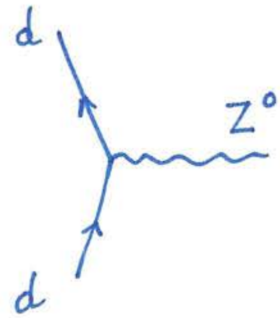
- FEYNMAN RULES : NEUTRAL CURRENT WEAK INTERACTION

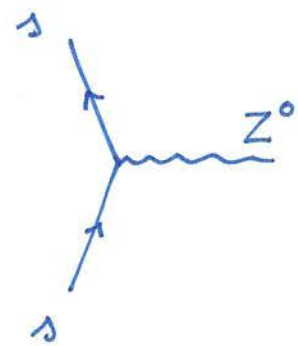




$$\boxed{-i\frac{g}{4\cos\theta_W}\gamma^\mu\left(\left[1-\frac{8}{3}\sin^2\theta_W\right]-\gamma_5\right)}$$

V/A ≈ 38%)





$$\boxed{+i\frac{g}{4\cos\theta_W}\gamma^\mu\left(\left[1-\frac{4}{3}\sin^2\theta_W\right]-\gamma_5\right)}$$

V/A ≈ 69%

∴ NC WEAK INTERACTION IS
FLAVOR CONSERVING !

⇒ HIGGS MECHANISM

- **PROBLEM**

$SU(2) \times U(1)_Y$ GAUGE SYMMETRY OF STANDARD ELECTROWEAK THEORY

↳ 4 GAUGE BOSONS: γ, W^+, W^-, Z^0

MANIFEST GAUGE SYMM REQUIRES GAUGE BOSONS TO BE MASSLESS

BUT EXPERIMENTALLY $M_W \simeq 80.4$ GeV

$M_Z \simeq 91.2$ GeV

⇒ ADDING MASS TERMS IN LAGRANGIAN YIELDS INCONSISTENT THEORY → EXPLICIT SYMM. BREAKING

UNRENORMALIZABLE DIVERGENCES APPEAR

⇒ HAVE TO INTRODUCE MASS WITHOUT BREAKING GAUGE INV.

SPONTANEOUS SYMMETRY BREAKING

- **SPONTANEOUS SYMMETRY BREAKING** (DISCRETE SYMMETRY)

CONSIDER SIMPLE EXAMPLE: SCALAR FIELD THEORY

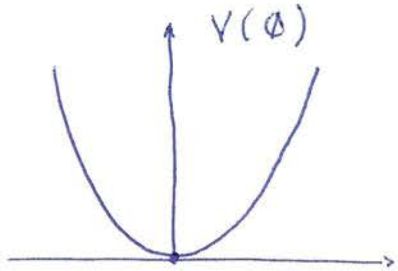
$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{1}{2}\mu^2\phi^2 - \underbrace{\frac{\lambda}{4}\phi^4}_{\text{INTERACTION TERM } (\lambda > 0)}$$

$$= T - V$$

$$V = \frac{1}{2}\mu^2\phi^2 + \frac{\lambda}{4}\phi^4$$

$\mathcal{L}$ HAS SYMMETRY w.r.t $\phi \to -\phi$

↳ IF $\mu^2 > 0$



⇓

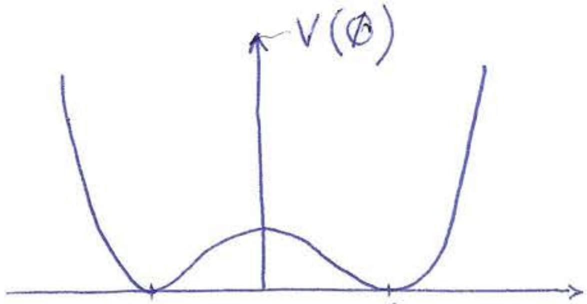
MINIMUM OF V

$\phi = 0$

$\Rightarrow$ GROUND STATE $(\phi = 0)$ REFLECTS THE SYMMETRY $(\phi \to -\phi)$ OF $\mathcal{L}$

∴ <u>MANIFEST</u> SYMMETRY (SYMM OF DYNAMICS = SYMM OF GROUND STATE)

↳ IF $\mu^2 < 0$



⇓

MINIMUM OF V

$$\frac{dV}{d\phi} = \mu^2 \phi + \lambda \phi^3 = \phi\left(\mu^2 + \lambda \phi^2\right) = 0$$

$\phi = 0$ MAXIMUM

$\phi = \pm \phi_0 = \pm\sqrt{-\frac{\mu^2}{\lambda}}$ MINIMUM

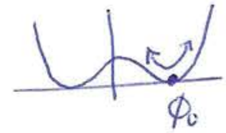
2 DEGENERATE MINIMA / GROUND STATES $\phi_0, -\phi_0$

∴ <u>HIDDEN</u> SYMMETRY (SYMM. OF DYNAMICS IS <u>NOT</u> SYMM OF GROUND STATE)

SPONTANEOUS BREAKING OF SYMMETRY

↳ PERTURBATIVE EXPANSION AROUND $\phi_0 = +\sqrt{-\frac{\mu^2}{\lambda}}$

$$\phi(x) = \phi_0 + \eta(x)$$



$$\mathcal{L}' = \frac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) - \frac{1}{2}\mu^2\eta^2 - \cancel{\mu^2\phi_0\eta}$$

$$- \frac{\lambda}{4}\left[\cancel{4\phi_0^3\eta} + 6\phi_0^2\eta^2 + 4\phi_0\eta^3 + \eta^4\right] + \text{Const.}$$

$$\mathcal{L}' = \frac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) - \frac{1}{2}\underbrace{(-2\mu^2)}_{\equiv m_\eta^2}\eta^2$$

$$- \lambda\phi_0\eta^3 - \frac{\lambda}{4}\eta^4$$

↳ WE HAVE GENERATED A MASS m_η
IN A MASSLESS THEORY
BY REQUIRING SPONTANEOUS SYMM BREAKING

↳ PHYSICAL EXAMPLES :

FERROMAGNETISM : SPINS CAN ALIGN ALONG ONE DIRECTION

DEGENERATE GROUND STATES

↑ ↑ ↑ ↑ ↑   ↓ ↓ ↓ ↓ ↓

• SPONTANEOUS BREAKING OF GLOBAL GAUGE SYMMETRY

↳ COMPLEX SCALAR FIELD ϕ , ϕ^*

$$\phi = \frac{1}{\sqrt{2}} (\phi_1 + i\phi_2)$$

$$\mathcal{L} = (\partial_\mu \phi)^* (\partial^\mu \phi) - \mu^2 \phi^* \phi - \lambda (\phi^* \phi)^2$$

$\mathcal{L}$ HAS GLOBAL GAUGE SYMM $U(1)$

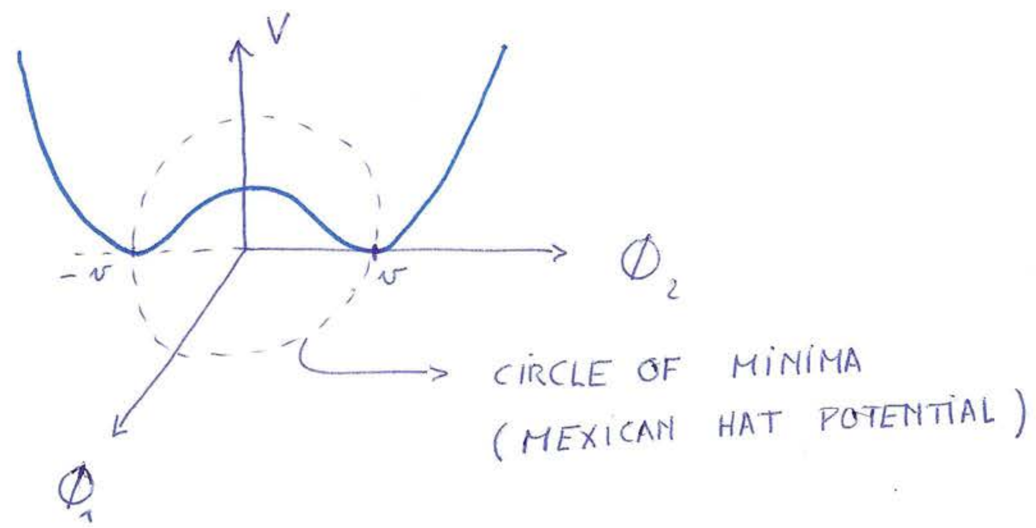
$$\phi \xrightarrow{U(1)} e^{i\alpha} \phi \qquad \text{(GLOBAL PHASE TF.)}$$

IN TERMS OF ϕ_1, ϕ_2

$$\mathcal{L} = \frac{1}{2} (\partial_\mu \phi_1)^2 + \frac{1}{2} (\partial_\mu \phi_2)^2$$

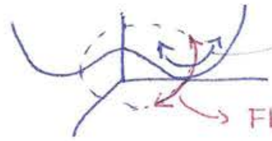
$$- \underbrace{\left[\frac{1}{2} \mu^2 (\phi_1^2 + \phi_2^2) + \frac{\lambda}{4} (\phi_1^2 + \phi_2^2)^2 \right]}_{V(\phi_1, \phi_2)}$$

FOR $\mu^2 < 0$, $\lambda > 0$: MINIMA OF V : $\phi_1^2 + \phi_2^2 = v^2 = -\frac{\mu^2}{\lambda}$



↳ PERTURBATIVE EXPANSION

$$\phi(x) = \frac{1}{\sqrt{2}} \left[v + \eta(x) + i\,\xi(x) \right]$$



$$v = \sqrt{-\frac{\mu^2}{\lambda}}$$

$$\mathcal{L}' = \frac{1}{2}(\partial_\mu \xi)^2 + \frac{1}{2}(\partial_\mu \eta)^2$$

$$+ \quad 0 \quad - \frac{1}{2} \underbrace{(-2\mu^2)}_{m_\eta^2 \,=\, 2v^2\lambda} \eta^2$$

$$+ \text{ CUBIC \& QUARTIC TERMS IN } \eta, \xi$$

η FIELD HAS ACQUIRED MASS $m_\eta = \sqrt{-2\mu^2}$

ξ FIELD IS MASSLESS

↓

FLUCTUATIONS ALONG RIM
(FROM ONE MINIMUM TO ANOTHER)
COST NO ENERGY

⇓

GOLDSTONE BOSONS

↳ PHYSICAL EXAMPLE : FERROMAGNETISM $H = -\bar{S}_i \cdot \bar{S}_j$

ROTATIONAL SYMM IS SPONTANEOUSLY BROKEN

↑ ↑ ↑ $\quad \curvearrowright \quad$ ↗ ↗ ↑ $\quad \longrightarrow \quad$ → →

↑ ↑ ↑ $\quad$ GOLDSTONE BOSONS: SPIN WAVES $\quad$ ↗ ↗ ↑ $\quad$ SPIN WAVE $\quad$ → →

- SPONTANEOUS SYMMETRY OF LOCAL GAUGE SYMMETRY (HIGGS MECHANISM) : $U(1)$ SYMMETRY

$$\mathcal{L}_0 = (\partial_\mu \phi)^* (\partial^\mu \phi) - \mu^2 \phi^* \phi - \lambda (\phi^* \phi)^2$$

REQUIRE $\mathcal{L}$ HAS $U(1)$ LOCAL GAUGE SYMM

$$\phi(x) \xrightarrow{U(1)} e^{i\alpha(x)} \phi(x)$$

$$D^\mu = \partial^\mu + ie A^\mu$$

$$A^\mu \xrightarrow{U(1)} A^\mu - \frac{1}{e} \partial^\mu \alpha$$

$$\mathcal{L} = \left[(\partial_\mu - ie A_\mu) \phi^* \right] \left[(\partial^\mu + ie A^\mu) \phi \right] - \mu^2 \phi^* \phi - \lambda (\phi^* \phi)^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

↳ FOR $\mu^2 > 0$: THIS IS QED LAGRANGIAN FOR PARTICLE WITH MASS μ + INTERACTION TERM $\lambda \phi^4$

↳ $\mu^2 < 0$: SPONTANEOUS SYMM BREAKING

$$\phi(x) = \frac{1}{\sqrt{2}}\left[v + \eta(x) + i\,\xi(x)\right]$$

$$\mathcal{L}' = \frac{1}{2}(\partial_\mu \xi)^2 + \frac{1}{2}(\partial_\mu \eta)^2 - \frac{1}{2}\underbrace{(2v^2\lambda)}_{m_\eta^2}\eta^2$$

$$+ \frac{1}{2} e^2 v^2 A_\mu A^\mu + e v A_\mu (\partial^\mu \xi) - \frac{1}{4} F_{\mu\nu}F^{\mu\nu}$$

+ INTERACTION TERMS

↳ GAUGE BOSON (A^μ FIELD)
HAS ACQUIRED A MASS $\boxed{m_A = e v}$!

BUT
- MASSIVE SPIN 1 PARTICLE HAS 3 POLARIZATIONS (2 TRANSVERSE + 1 LONGITUDINAL)
- MASSLESS SPIN 1 PARTICLE HAS ONLY 2 TRANSVERSE POLARIZATIONS

↳ GAUGE TF. TO ELIMINATE UNPHYSICAL FIELDS ?

$$\phi(x) \simeq \frac{1}{\sqrt{2}}\left(v + \eta(x)\right) e^{i\xi/v}$$ TO LOWEST ORDER IN ξ

GAUGE TF

$$\phi \to \frac{1}{\sqrt{2}}\left(v + h(x)\right) e^{\frac{i}{v}\theta(x)}$$

$$A^\mu \to A^\mu - \frac{1}{e}\frac{1}{v}\partial^\mu \theta(x)$$

WORK WITH h & θ , A^μ FIELDS

$$(\partial^\mu + ie A^\mu)\, \phi$$

$$= \frac{1}{\sqrt{2}}\, e^{\frac{i}{v}\theta(x)} \left(\partial^\mu h + ie\,(v+h)\, A^\mu \right)$$

$$\mathcal{L}'' = \frac{1}{2}(\partial_\mu h)^2 - \lambda v^2 h^2 - \lambda v h^3 - \frac{1}{4}\lambda h^4$$
$$+ \frac{1}{2} e^2 v^2 A^\mu A_\mu + e^2 v A^\mu A_\mu h + \frac{1}{2} e^2 A^\mu A_\mu h^2$$
$$- \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

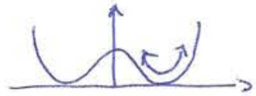
GOLDSTONE BOSON (ξ FIELD) DOES NOT APPEAR IN THEORY) HIGGS MECHANISM

HAS BEEN TRANSFORMED AWAY

PHYSICALLY : DEGREE OF FREEDOM CORRESPONDS WITH LONGITUDINAL POL. OF A^μ FIELD

$\Rightarrow$ GAUGE BOSONS HAVE ACQUIRED MASS

$\underline{m_A = e v}$

$\Rightarrow$ EXCITATIONS : h 

$\Downarrow$

HIGGS FIELD : MASS $\underline{m_h^2 = 2\lambda v^2}$

(MASSIVE SCALAR PARTICLE)