Theoretical Elementary Particle Physics
Exercise 6

10 January 2019

1 General idea (10 points)

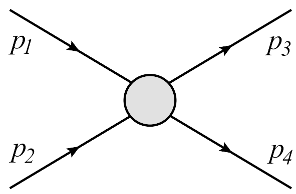
1.1 Mandelstam variables (5 points)

Show that the n-point amplitude depends on $(3n-10)$ independent Mandelstam variables.

1.2 Subtractions (5 points)

Write down the dispersive representation for $\log(s)$. **Hint:** at least one subtraction is necessary.

2 Elastic scattering (20 points)



Consider $2 \to 2$ elastic scattering of the particles with mass m. The partial wave decomposition for the amplitude is then

$$A(s,t) = \sum_{l=0}^{\infty}(2l+1)a_l(s)P_l(\cos\theta)\,, \quad s = (p_1+p_2)^2\,, \tag{1}$$

where θ is the scattering angle, P_l are Legendre polynomials and unitarity relation

$$\mathrm{Im}\; a_l = \rho(s)|a_l|^2\,, \quad s \geq 4m^2\,. \tag{2}$$

(a) **(10 points)** Using unitarity constraint, show that elastic partial wave amplitude can be written in terms of one function: the phase shift $\delta_l(s)$.

(b) **(10 points)** Derive the unitarity relation for the inverse partial wave amplitude.

3 Triangle diagramm (70 points)

Consider simple triangle diagram shown on Fig. 1, where all particles are scalars and $q_1^2 = q_2^2 = 0$.

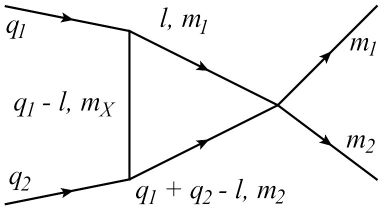


Figure 1: The triangle diagram for the scalar theory

(a) **(30 points)** Write down the expression for the amplitude $T(s)$ and evaluate the loop integral explicitly.

(b) **(40 points)** Calculate the discontinuity of the amplitude using the cutting rules and restore the full amplitude as a dispersive integral over the cut in the complex s-plane.