Theoretical Elementary Particle Physics
Exercise 4

3 December 2018

The Exercise 3 has to be handed on Monday, December 10. The Exercise 4 has to be handed on Thursday, December 13.

1 Gluon self-energy (100 points)

One-loop contributions of the gluon self energy (2 gluon, 1 ghost and 1 fermion loop) are shown on Fig. 1.

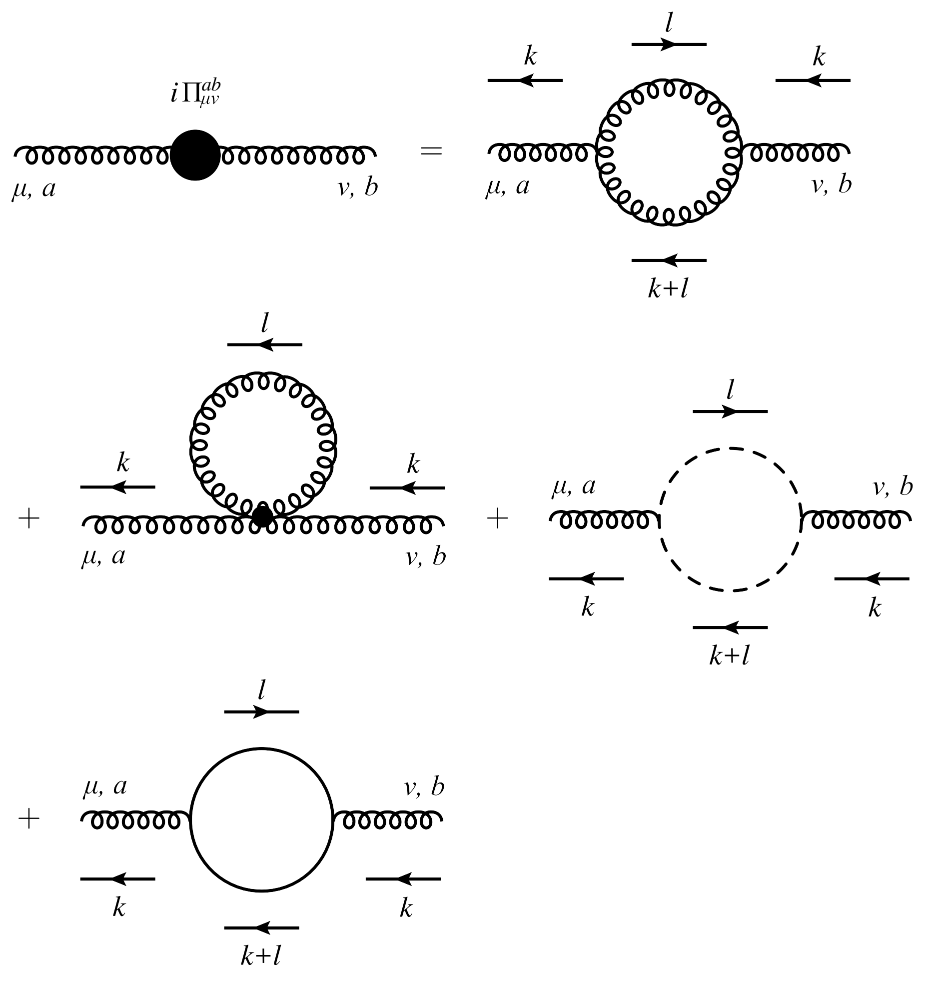

Figure 1: One-loop contributions of the gluon self energy. The first two diagrams are due to the triple and quadratic gluon coupling, respectively. The third diagram is a ghost loop and the last one represents a fermion loop. The 4-momenta are labelled on each diagram. Color indices a and b are also shown.

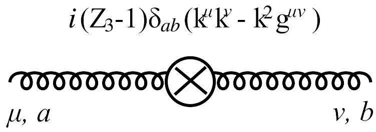

Figure 2. Counterterm that absorbs divergence of the diagrams in Fig. 1 together with the corresponding Feynman rule.

Because of the gauge invariance

$$k_\mu \Pi^{\mu\nu}_{ab}(k) = 0\,, \tag{1}$$

$\prod^{\mu\nu}_{ab}(k)$ can be written in the following form

$$\Pi^{\mu\nu}_{ab}(k) = \delta_{ab}(k^\mu k^\nu - k^2 g^{\mu\nu})\Pi(k^2)\,, \tag{2}$$

It is convenient to directly compute $\Pi(k^2)$ which is in 4 space-time dimensions expressed as:

$$i\Pi(k^2) = -\frac{1}{3k^2} g_{\mu\nu} i\Pi^{\mu\nu}_{aa}(k) \tag{3}$$

where k stands for external 4-momentum. The idea is to use the Feynman rules to obtain 1-loop contribution for each diagram in Fig. 1. By doing so, one gets expressions $i\Pi^{j,\mu\nu}_{aa}(k)$ where the color index b is replaced by the index a because of the trivial δ_{ab}, and j takes values from 1 to 4 since there are 4 different 1-loop contributions. The total one-loop correction to the gluon propagator is obtained by adding all 4 contributions:

$$i\Pi^{\mu\nu}_{aa}(k) = \sum_{j=1}^{4} i\Pi^{j,\mu\nu}_{aa}(k) \tag{4}$$

(a) **(20 points)** Write down one-loop contributions of the gluon self energy (gluon loops, ghost loop and quark loop contributions) in Feynman gauge $\xi = 1$. The Feynman rules for triple, quartic gluon couplings and the ghost field may be found in the lectures. In the diagram with the ghost loop additional $(-)$ sign should be included. In the case of a fermion loop in addition to the $(-)$ sign, trace of the integrand should be applied. Also use N_f as multiplicative factor because there is in principle N_f different diagrams with different quark flavors propagating in the loop. In the Standard model, $N_f = 6$ corresponding to $u, d, c, s, b,$ and t quarks. Notice that even though all these particles have different masses that enter into propagators, the divergent part is mass independent and N_f can be added as multiplicative factor for the purpose of renormalization.

Note also that the two diagrams with the gluon loop have a symmetry factor equal to 2.

(b) **(70 points)** Using dimensional regularization, extract the divergent part (in $1/\epsilon$) for the gluon loop contribution, the ghost loop contribution and the quark loop contribution separately. **Hint**: Start by applying Eq. 3.

After employing Feynman parametrization bypass the Wick rotation by directly applying formulae

$$\int \frac{d^d l}{(2\pi)^d} \frac{1}{(l^2-\Delta)^n} = \frac{(-1)^n i}{(4\pi)^{d/2}} \frac{\Gamma(n-\frac{d}{2})}{\Gamma(n)} \left(\frac{1}{\Delta}\right)^{n-\frac{d}{2}} \tag{5}$$

$$\int \frac{d^d l}{(2\pi)^d} \frac{l^2}{(l^2-\Delta)^n} = \frac{(-1)^{n-1} i}{(4\pi)^{d/2}} \frac{d}{2} \frac{\Gamma(n-\frac{d}{2}-1)}{\Gamma(n)} \left(\frac{1}{\Delta}\right)^{n-\frac{d}{2}-1} \tag{6}$$

Use $d = 4 - 2\epsilon$ which implies the following expansion of the Γ function around pole $z = -1$

$$\Gamma(-1+\epsilon) \approx -\frac{1}{\epsilon} + \gamma - 1\,, \tag{7}$$

and $z = 0$

$$\Gamma(\epsilon) \approx \frac{1}{\epsilon} - \gamma\,. \tag{8}$$

(c) (10 points) Absorbing this divergence into the gluon field renormalization (see Fig. 2) constant Z_3, show that the one-loop contribution to the gluon renormalization constant is given by

$$Z_3 = 1 - \frac{g^2}{(4\pi)^2}\frac{1}{\epsilon}\left[-5 + \frac{2}{3}N_f\right]. \tag{9}$$