In recent years the ab initio treatment of the nuclear many-body problem has seen tremendous progress such that the A-body Schrödinger equation can be solved from first principles using realistic nuclear Hamiltonians with a sound link to QCD. However, it is yet unclear (1) how to describe genuine open-shell nuclei from an ab initio perspective and (2) how to overcome the curse of dimensionality in many-body approaches that prevents relaxing many-body approximations. Additionally, the large uncertainties arising from the input Hamiltonian make a direct comparison with experimental observations challenging, requiring extensive interaction benchmarks far away from shell closures and for large mass numbers in the future. Exploiting symmetry breaking enables for addressing nuclear observables in arbitrary open-shell systems. The recently introduced Bogoliubov extension of many-body perturbation theory serves as an example for a computationally light-weighted approach well-suited for benchmarking ground-state energetics along medium-mass isotopic chains. In a complementary way, tensor-decomposition techniques help resolving the computational bottlenecks one is facing in state-of-the-art many-body implementations. By factorizing many-body tensors, like the Hamiltonian, storage requirements are lowered and, at the same time, the contraction pattern of the tensor network is optimized ultimately yielding a reformulation of many-body theory as it stands today. The combination of novel many-body expansions and innovative tools from applied mathematics allow for extending the range of ab initio applications and, thus, putting the next generation of nuclear Hamiltonians to a stringent test at low computational cost.