Handout 7 (read by Dec. 4)

Guide for constructing a character table

Let G be a finite group of order $|G|$. The information contained in the characters of irreducible representations may be summarized in terms of a so-called character table. In the following we summarize the results of the previous section and provide a guide for constructing the character table.

1. A character table is a quadratic matrix, whose entries consist of the values of the characters of the non-equivalent irreducible representations. A row contains the values for a given irreducible representation, whereas the conjugacy classes are sorted in terms of the columns.

2. The total number r of non-equivalent irreducible representations equals the number k of conjugacy classes (theorem 2.3.12). Therefore, the character table is an $r \times r$ matrix:

	K_1	K_2	$\ldots$	K_r
$\chi^{(1)}$	x	x	$\ldots$	x
$\chi^{(2)}$	x	x	$\ldots$	x
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\chi^{(r)}$	x	x	$\ldots$	x

3. The dimensions n_μ ($\mu = 1, \ldots, r$) of the non-equivalent irreducible representations satisfy the following sum rule related to the order $|G|$ of the group (theorem 2.3.16):
$$\sum_{\mu=1}^{r} n_\mu^2 = |G|.$$

4. For any group G, the trivial representation, $D^{(1)}(g) = 1 \; \forall \; g \in G$, always exists. By convention, we denote this representation with the label 1 with $n_1 = 1$.

5. The value of the character of the unit element e is given by the dimension of the corresponding representation:
$$\chi^{(\mu)}(e) = n_\mu.$$
The unit element is only conjugate to itself, i.e., $(e) = \{e\}$. We always denote the corresponding conjugacy class by $K_1 = (e)$.

6. For *one-dimensional* representations we have $\chi(g) = D(g)$ such that
   (a) $\chi(g_1 g_2) = \chi(g_1)\chi(g_2)$,
   (b) $\chi(g) \neq 0$.

7. The characters of non-equivalent irreducible representations satisfy the orthogonality relation (theorem 2.3.10)
$$\frac{1}{|G|} \sum_{i=1}^{k} k_i \chi_i^{(\mu)} \chi_i^{(\nu)*} = \delta^{\mu\nu},$$
where k_i is the number of group elements in the conjugacy class K_i.

8. Concerning the conjugacy classes the orthogonality relation reads (theorem 2.3.18)

$$\frac{k_i}{|G|} \sum_{\mu=1}^{r} \chi_i^{(\mu)} \chi_j^{(\mu)*} = \delta_{ij}.$$

9. Make use of any additional 'useful' information.