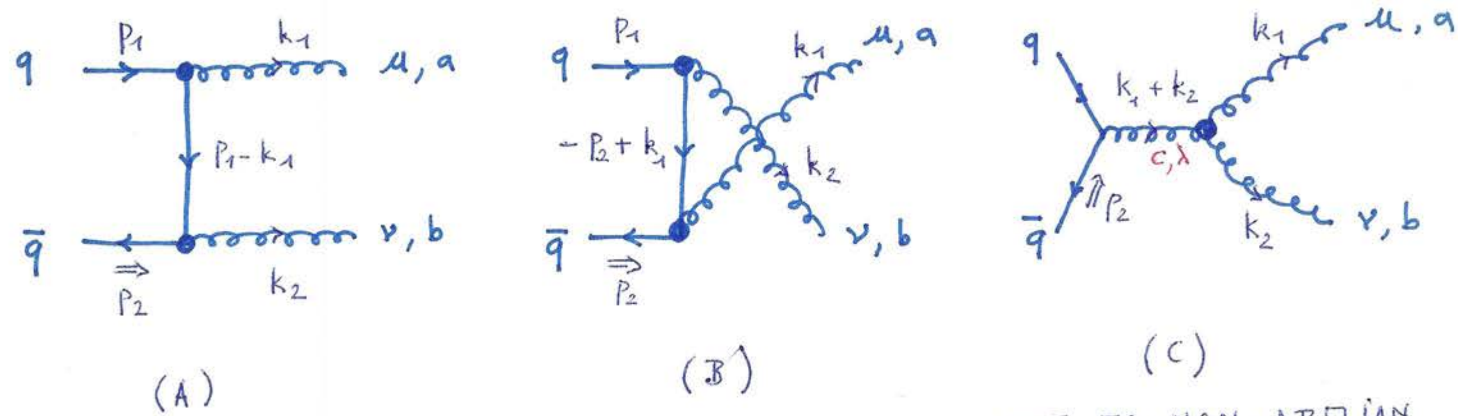
# QUANTIZATION OF A NON-ABELIAN GAUGE THEORY (QCD)

⇒ **GAUGE INVARIANCE**

e.g. 2 GLUON PROCESS $q\bar{q} \to gg$




AS IN QED

DUE TO NON-ABELIAN NATURE OF GAUGE GROUP SU(3)

- $$\left(\mathcal{M}_A^{\mu\nu}\right)_{ab} = \bar{v}(p_2, s_2)\left[-ig\frac{\lambda_b}{2}\gamma^\nu\right]\frac{i(\not{p}_1 - \not{k}_1 + m)}{(p_1-k_1)^2 - m^2}\left[-ig\frac{\lambda_a}{2}\gamma^\mu\right]u(p_1, s_1)$$

$$= -ig^2\frac{\lambda_b}{2}\frac{\lambda_a}{2}\,\bar{v}(p_2, s_2)\,\gamma^\nu\frac{(\not{p}_1 - \not{k}_1 + m)}{(p_1-k_1)^2 - m^2}\gamma^\mu\,u(p_1, s_1)$$

- $$\left(\mathcal{M}_B^{\mu\nu}\right)_{ab} = -ig^2\frac{\lambda_a}{2}\frac{\lambda_b}{2}\,\bar{v}(p_2, s_2)\,\gamma^\mu\frac{(-\not{p}_2 + \not{k}_1 + m)}{(-p_2+k_1)^2 - m^2}\gamma^\nu\,u(p_1, s_1)$$

- PHYSICAL AMPLITUDE

$$\mathcal{M}_{ab} = \mathcal{M}_{ab}^{\mu\nu} \cdot \varepsilon_\mu^*(k_1, \lambda_1)\ \varepsilon_\nu^*(k_2, \lambda_2)$$

GLUON POLARIZATION VECTORS

PHYSICAL GLUONS ARE TRANSVERSE ($\lambda_1 = \pm 1$)

$$\Rightarrow k_1^2 = 0, \quad k_1^\mu \cdot \varepsilon_\mu(k_1, \lambda_1 = \pm 1) = 0$$

$$\& \quad k_2^2 = 0, \quad k_2^\nu \cdot \varepsilon_\nu(k_2, \lambda_2 = \pm 1) = 0$$

GAUGE INVARIANCE : $\varepsilon_\mu(k_1, \lambda_1) \to \varepsilon_\mu(k_1, \lambda_1) - a(k_1)_\mu$

$$\Downarrow$$

$$(k_1)_\mu \cdot \mathcal{M}_{ab}^{\mu\nu} = 0$$

ANALOGOUSLY $(k_2)_\nu\ \mathcal{M}_{ab}^{\mu\nu} = 0$

- CHECK OF GAUGE INVARIANCE

$$\rightarrow (k_1)_\mu \left(\mathcal{M}_A^{\mu\nu}\right)_{ab} = -ig^2 \frac{\lambda_b}{2} \frac{\lambda_a}{2}\ \bar{v}(p_2, s_2)\ \gamma^\nu \frac{(\not{p}_1 - \not{k}_1 + m)}{(p_1 - k_1)^2 - m^2}\ \not{k}_1\ u(p_1, s_1)$$

$$\not{k}_1\ u(p_1, s_1) = -(\not{p}_1 - \not{k}_1 - m)\ u(p_1, s_1)$$

$$= +ig^2 \frac{\lambda_b}{2} \frac{\lambda_a}{2}\ \bar{v}(p_2, s_2)\ \gamma^\nu\ u(p_1, s_1)$$

$$\rightarrow (k_1)_\mu \left(\mathcal{M}_B^{\mu\nu}\right)_{ab} = -ig^2 \frac{\lambda_a}{2} \frac{\lambda_b}{2}\ \bar{v}(p_2, s_2)\ \not{k}_1\ \frac{(-\not{p}_2 + \not{k}_1 + m)}{(-p_2 + k_1)^2 - m^2}\ \gamma^\nu\ u(p_1, s_1)$$

$$\bar{v}(p_2, s_2)\ \not{k}_1 = \bar{v}(p_2, s_2)\ (-\not{p}_2 + \not{k}_1 - m)$$

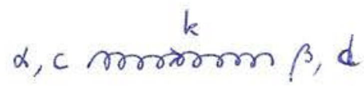
$$= -ig^2 \frac{\lambda_a}{2} \frac{\lambda_b}{2}\ \bar{v}(p_2, s_2)\ \gamma^\nu\ u(p_1, s_1)$$

$$\rightarrow \boxed{\begin{aligned} &(k_1)_\mu \left(\mathcal{M}_A^{\mu\nu} + \mathcal{M}_B^{\mu\nu}\right)_{ab} \\ &= -ig^2 \underbrace{\left[\frac{\lambda_a}{2}, \frac{\lambda_b}{2}\right]_-}_{i f_{abc} \frac{\lambda_c}{2}} \bar{v}(p_2, s_2)\, \gamma^\nu\, u(p_1, s_1) \\ &= g^2 f_{abc} \frac{\lambda_c}{2} \bar{v}(p_2, s_2)\, \gamma^\nu\, u(p_1, s_1) \end{aligned}}$$

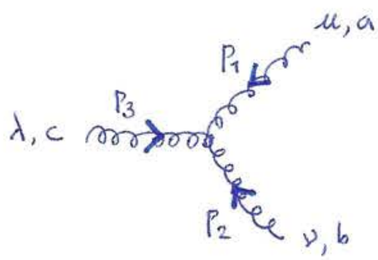
$\nearrow$ QED ("$f_{abc} = 0$") : $(k_1)_\mu \cdot \left(\mathcal{M}_A^{\mu\nu} + \mathcal{M}_B^{\mu\nu}\right) = 0$

$\searrow$ QCD ($f_{abc} \neq 0$) : $(k_1)_\mu \cdot \left(\mathcal{M}_A^{\mu\nu} + \mathcal{M}_B^{\mu\nu}\right) \neq 0$

- IN QCD CASE WE HAVE TO ADD DIAGRAM C




FEYNMAN GAUGE : $\dfrac{-i g^{\alpha\beta}}{k^2} \cdot \delta_{cd}$




$$- g f_{abc} \left\{ g^{\mu\nu} (p_1 - p_2)^\lambda + g^{\nu\lambda} (p_2 - p_3)^\mu + g^{\lambda\mu} (p_3 - p_1)^\nu \right\}$$

$$\left(\mathcal{M}_c^{\mu\nu}\right)_{ab} = \bar{v}(p_2, s_2)\left[-ig\frac{\lambda_c}{2}\gamma_\lambda\right]u(p_1, s_1)$$

$$\cdot \frac{(-i)}{(k_1+k_2)^2}$$

$$\cdot(-g f_{abc})\left\{g^{\mu\nu}(-k_1+k_2)^\lambda + g^{\nu\lambda}(-k_1-2k_2)^\mu + g^{\lambda\mu}(2k_1+k_2)^\nu\right\}$$

$$\longrightarrow (k_1)_\mu\left(\mathcal{M}_c^{\mu\nu}\right)_{ab} = g^2 f_{abc}\frac{\lambda_c}{2}\,\bar{v}(p_2, s_2)\gamma_\lambda u(p_1, s_1)$$

$$\cdot\frac{1}{2k_1\cdot k_2}\left\{k_1^\nu(-k_1+k_2)^\lambda + g^{\nu\lambda}(-2k_1\cdot k_2) + k_1^\lambda(2k_1+k_2)^\nu\right\} \qquad (k_1^2=0)$$

$$= g^2 f_{abc}\frac{\lambda_c}{2}\cdot\frac{1}{2k_1\cdot k_2}\cdot\bar{v}(p_2, s_2)\left\{k_1^\nu(\not{k}_1+\not{k}_2) + \not{k}_1 k_2^\nu - 2k_1\cdot k_2\,\gamma^\nu\right\}u(p_1, s_1)$$

$$\downarrow$$

$$\bar{v}(p_2, s_2)(\not{k}_1+\not{k}_2)u(p_1, s_1)$$

$$= \bar{v}(p_2, s_2)(\not{p}_2+\not{p}_1)u(p_1, s_1)$$

$$= \bar{v}(p_2, s_2)(-m+m)u(p_1, s_1) = 0$$

$$\therefore \boxed{(k_1)_\mu\left(\mathcal{M}_c^{\mu\nu}\right)_{ab} = -g^2 f_{abc}\frac{\lambda_c}{2}\,\bar{v}(p_2, s_2)\left\{\gamma^\nu - \frac{k_2^\nu\not{k}_1}{2k_1\cdot k_2}\right\}u(p_1, s_1)}$$

- SUM OF A + B + C

$$\boxed{\begin{aligned}&(k_1)_\mu \left(\mathcal{M}_A^{\mu\nu} + \mathcal{M}_B^{\mu\nu} + \mathcal{M}_C^{\mu\nu}\right)_{ab} \\ &= g^2 f_{abc} \frac{\lambda_c}{2} \bar{v}(p_2, s_2) \frac{k_2^\nu \not{k}_1}{2 k_1 . k_2} u(p_1, s_1)\end{aligned}}$$

$\downarrow$

ZERO WHEN CONTRACTING WITH

REAL GLUON POLARIZATION VECTOR $\varepsilon^*_\nu(k_2, \lambda_2)$

BECAUSE $(k_2)^\nu_\cdot \, \varepsilon_\nu(k_2, \lambda_2) = 0$ $\qquad \lambda_2 = \pm 1$

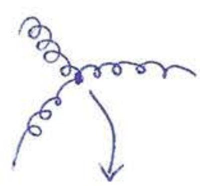
$\therefore$ DIAGRAM C IS NEEDED IN QCD CASE

TO PRESERVE GAUGE INVARIANCE

$\downarrow$

CANCELLATION $A + B \Leftrightarrow C$

REQUIRES THAT STRONG COUPLING CONSTANT

IN  AND 

$g \qquad\qquad g$

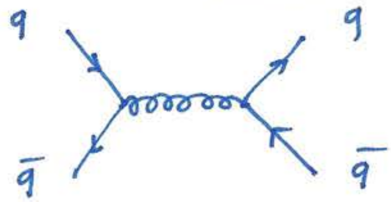
ARE SAME !

$\hookrightarrow$ ANALOGOUSLY $(k_2)_\nu \left(\mathcal{M}_A^{\mu\nu} + \mathcal{M}_B^{\mu\nu} + \mathcal{M}_C^{\mu\nu}\right) . (\varepsilon_1)_\mu = 0$

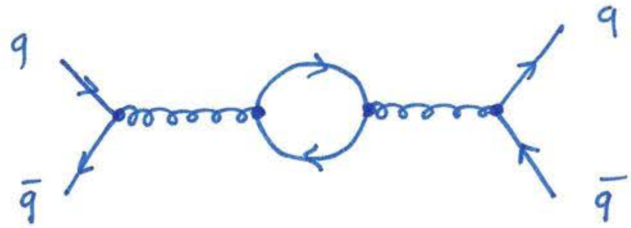
⇒ LOOPS

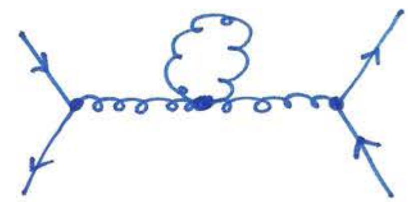
PROCESS $q\bar{q} \to q\bar{q}$ WITH $2g$ (VIRTUAL) INTERMEDIATE STATE

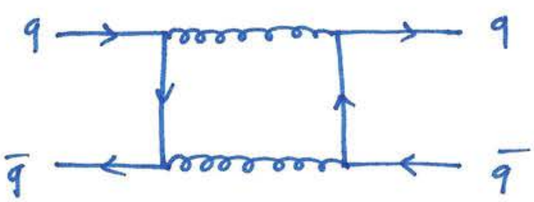
- $q\bar{q} \to q\bar{q}$ TO ORDER $g^2$



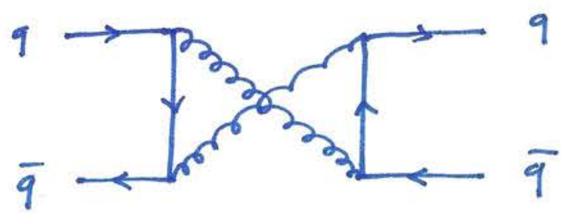

- $q\bar{q} \to q\bar{q}$ TO ORDER $g^4$






(A)

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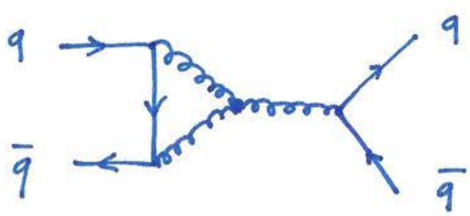


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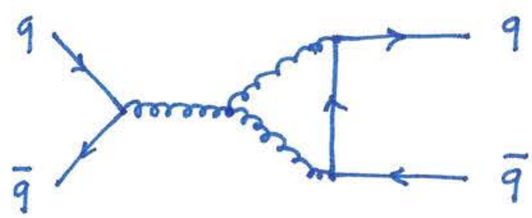



(D)

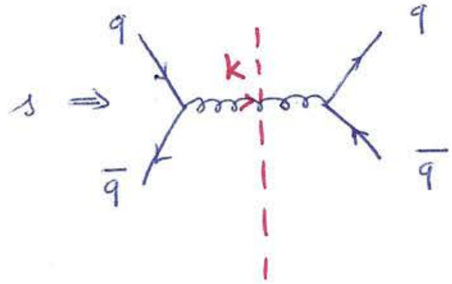
(E)

- UNITARITY ('CONSERVATION' OF PROBABILITY)

CALCULATE 'ABSORPTIVE' PART (IMAGINARY PART)

OF $q\bar{q} \to q\bar{q}$

↳ TO ORDER $g^2$




TREE LEVEL

↓

REAL

↓

NO ABSORPTIVE PART

ABS PART

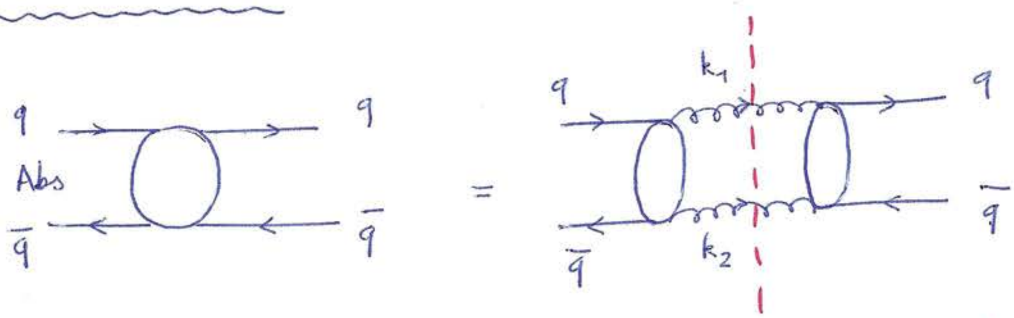
⇕

PUT INTERMEDIATE STATE ON-SHELL ($k^2 = 0$)

GLUON CANNOT BE PUT ON SHELL

IN PHYSICAL REGION AS $s = (P_1 + P_2)^2 > 4m^2 > 0$

↳ TO ORDER $g^4$




CAN BE PUT ON-SHELL

⇓ $(k_1^2 = k_2^2 = 0)$

ABSORPTIVE PART

## ↳ UNITARITY IN GENERAL

$|i\rangle$  $|f\rangle$

SCATTERING PROCESS : DESCRIBED BY S-MATRIX ELEMENT $S_{fi}$

S-MATRIX IS UNITARY (TOTAL PROBABILITY OF SCATTERING IS 1)

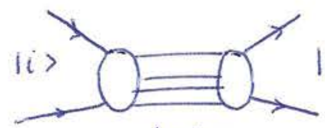
$$S^{+} S = \mathbb{1}$$

$$S = \mathbb{1} - i\,T$$

↳ T-MATRIX ⟺ SCATTERING AMPLITUDE

$$(S^{+} S)_{fi} = \delta_{fi}$$

$$(S^{+})_{fn}\,(S)_{ni} = \delta_{fi}$$

$|i\rangle$  $|f\rangle$

$|n\rangle$

$$S^{*}_{nf}\,S_{ni} = \delta_{fi}$$

$\sum_n |n\rangle\langle n|$ : COMPLETE SET OF INTERMEDIATE STATES

$$(1 + i\,T^{*})_{nf}\,(1 - i\,T)_{ni} = \delta_{fi}$$

$$\delta_{fi} - i\,T_{fi} + i\,T^{*}_{if} + T^{*}_{nf}\,T_{ni} = \delta_{fi}$$

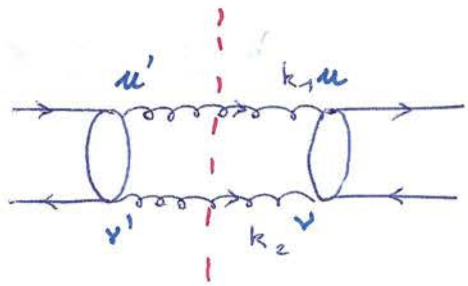
$$\boxed{i\,(T_{fi} - T^{*}_{if}) = (T^{+})_{fn}\,(T)_{ni}}$$

FOR $|i\rangle = |f\rangle$ ELASTIC PROCESS

$$i\,\underbrace{(T_{ii} - T^{*}_{ii})}_{2i\,\mathrm{Im}\,T_{ii}} = (T^{+})_{in}\,(T)_{ni}$$

$$\boxed{\mathrm{Abs}\,T_{ii} \equiv +\,2\,\mathrm{Im}\,T_{ii} = -\,(T^{+})_{in}\,(T)_{ni}}$$

## ↳ $q\bar{q} \to q\bar{q}$ TO ORDER $g^4$




$|n\rangle$ : 2 GLUON INTERMEDIATE STATE

$$\sum_n \iff \frac{1}{2} \int d\rho^{(2)} \sum_{a,b} \sum_{\lambda_1 = \pm 1} \sum_{\lambda_2 = \pm 1} = \frac{1}{2} \sum_{a,b} \sum_{\lambda_1 = \pm 1} \int \frac{d^3 \vec{k}_1}{(2\pi)^3 2|\vec{k}_1|} \sum_{\lambda_2 = \pm 1} \int \frac{d^3 \vec{k}_2}{(2\pi)^3 2|\vec{k}_2|}$$

2 IDENTICAL GLUONS IN INTERMEDIATE STATE

PHASE SPACE OF 2-GLUON STATE

BOTH GLUONS ARE TRANSVERSE (PHYSICAL d.o.f.)

(1) RHS OF UNITARITY EQUATION

$$\text{Abs}\, T^{\text{(PHYS)}}_{q\bar{q} \to q\bar{q}} = -\frac{1}{2} \int d\rho^{(2)} \sum_{\lambda_1 = \pm 1} \sum_{\lambda_2 = \pm 1} \left( T^{+\mu\nu}_{ab}\, \varepsilon_\mu(1)\, \varepsilon_\nu(2) \right) \left( T^{\mu'\nu'}_{ab}\, \varepsilon^*_{\mu'}(1)\, \varepsilon^*_{\nu'}(2) \right)$$

$$\sum_{\lambda_1 = \pm 1} \varepsilon_\mu(1)\, \varepsilon^*_{\mu'}(1) \equiv (-g_T)_{\mu\mu'}$$

PROJECTOR FOR TRANSVERSE GLUONS

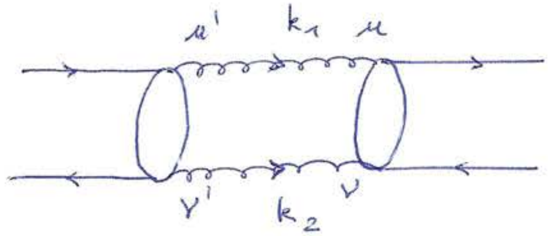
$$\sum_{\lambda_2 = \pm 1} \varepsilon_\nu(2)\, \varepsilon^*_{\nu'}(2) \equiv (-g_T)_{\nu\nu'}$$

$$\therefore \quad \boxed{\text{Abs}\, T^{\text{PHYS}}_{q\bar{q} \to q\bar{q}} = -\frac{1}{2} \int d\rho^{(2)}\, T^{+\mu\nu}_{ab} (-g_T)_{\mu\mu'} (-g_T)_{\nu\nu'}\, T^{\mu'\nu'}_{ab}}$$

$q\bar{q} \to 2g$ AMPL.

## (2) LHS OF UNITARITY EQUATION

$$\mathrm{Abs}\, T^{\mathrm{LOOP}}_{q\bar{q}\to q\bar{q}} = 2\, \mathrm{Im}\, T_{q\bar{q}\to q\bar{q}}$$




IN FEYNMAN DIAGRAM

FOR EACH INTERMEDIATE GLUON (FEYNMAN GAUGE)

$$\int \frac{d^4 k_1}{(2\pi)^4} \frac{i(-g_{\mu\mu'})}{k_1^2 + i\varepsilon}$$

$\downarrow$ PUT GLUON ON-SHELL (ABS. PART = 2 IM PART)

$$\int \frac{d^4 k_1}{(2\pi)^4} \, i(-g_{\mu\mu'}) \, (-2\pi i)\, \delta(k_1^2)\, \underbrace{\Theta(k_1^0)}_{\text{POS. ENERGY PART}}$$

$$= (-g_{\mu\mu'}) \int \frac{d^4 k_1}{(2\pi)^4} \frac{2\pi}{2|\vec{k}_1|} \delta(k_1^0 - |\vec{k}_1|)$$

$$= (-g_{\mu\mu'}) \underbrace{\int \frac{d^3 \vec{k}_1}{(2\pi)^3\, 2|\vec{k}_1|}}_{\text{PHASE SPACE INTEGRAL FOR 1 GLUON}}$$

$$\boxed{\begin{aligned} &\mathrm{Abs}\ T^{\text{LOOP}}_{q\bar{q}\to q\bar{q}} \\ &= -\frac{1}{2}\int d\rho^{(2)}\ T^{+\,\mu\nu}_{ab}\left(-g_{\mu\mu'}\right)\left(-g_{\nu\nu'}\right)\ T^{\mu'\nu'}_{ab}\end{aligned}}$$

FOR A THEORY WHICH RESPECTS UNITARITY

BOTH WAYS TO CALCULATE Abs T SHOULD BE EQUAL !

↳ IN GLUON PROPAGATOR NUMERATOR $\left(-g_{\mu\mu'}\right)$

THERE IS A SUM OVER BOTH T & L GLUON POLARIZATIONS

CONSIDER c.m. SYSTEM OF $2g$ SYSTEM


$$(k_1+k_2)^2 = s = 2\,k_1\cdot k_2$$

$$|\vec{k}_1| = |\vec{k}_2| = \frac{\sqrt{s}}{2}$$

DEFINE UNIT VECTORS (SUDAKOV VECTORS)

$(1,0,0,1)$ $\quad \hat{k}_1^{\mu} \equiv \dfrac{2}{\sqrt{s}}\,k_1^{\mu}$

$\rightarrow \hat{k}_1\cdot\hat{k}_2 = 2$

$(1,0,0,-1)$ $\quad \hat{k}_2^{\nu} \equiv \dfrac{2}{\sqrt{s}}\,k_2^{\nu}$

WE CAN USE THESE UNIT VECTORS TO DEFINE LONG. POL.

$$\boxed{-g^{\mu\mu'} = \sum_{\lambda_1=\pm1}\varepsilon^{\mu}(k_1,\lambda_1)\,\varepsilon^{\mu'*}(k_1,\lambda_1) - \frac{1}{2}\left(\hat{k}_1^{\mu}\hat{k}_2^{\mu'} + \hat{k}_1^{\mu'}\hat{k}_2^{\mu}\right)}$$

CHECK :

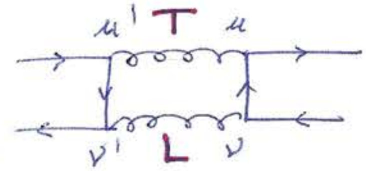
$$-g^{\mu\mu'}\,(k_1)_{\mu} = 0 - \frac{\sqrt{s}}{2}\,\frac{1}{2}\Big(\underbrace{\hat{k}_1^{2}}_{0}\,\hat{k}_2^{\mu'} + \hat{k}_1^{\mu'}\underbrace{\hat{k}_1\cdot\hat{k}_2}_{\underbrace{\frac{4}{s}\cdot\frac{s}{2}}_{2}}\Big)$$

$$-(k_1)^{\mu'} \stackrel{!}{=} (-k_1)^{\mu'}$$

$$\therefore \quad \boxed{-g^{\mu\mu'} = \underset{\uparrow \atop \text{T POLARIZATION}}{(-g_T)^{\mu\mu'}} - \underset{\uparrow \atop \text{L. POLARIZATION}}{\frac{1}{2}\left(\hat{k}_1^{\mu}\hat{k}_2^{\mu'} + \hat{k}_1^{\mu'}\hat{k}_2^{\mu}\right)}}$$

$\Delta T$ SHOULD BE 0 FOR UNITARY THEORY

- $\Delta T \equiv \text{Abs}\, T^{PHYS}_{q\bar{q}\to q\bar{q}} - \text{Abs}\, T^{LOOP}_{q\bar{q}\to q\bar{q}}$

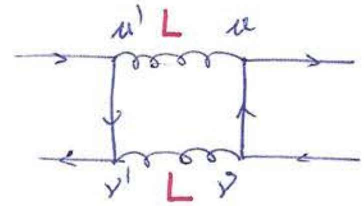



$$= \frac{1}{2}\int d\rho^{(2)}\; T^{+\mu\nu}_{ab} \Big\{ (-g_T)_{\mu\mu'} \left(-\tfrac{1}{2}\right)\left(\hat{k}_{1\nu}\underline{\hat{k}_{2\nu'}} + \hat{k}_{1\nu'}\underline{\hat{k}_{2\nu}}\right)$$

$$+ \left(-\tfrac{1}{2}\right)\left(\underline{\hat{k}_{1\mu}}\hat{k}_{2\mu'} + \underline{\hat{k}_{1\mu'}}\hat{k}_{2\mu}\right)(-g_T)_{\nu\nu'}$$

$$+ \left(-\tfrac{1}{2}\right)\left(\hat{k}_{1\mu}\hat{k}_{2\mu'} + \hat{k}_{1\mu'}\hat{k}_{2\mu}\right)$$

$$\cdot\left(-\tfrac{1}{2}\right)\left(\hat{k}_{1\nu}\hat{k}_{2\nu'} + \hat{k}_{1\nu'}\hat{k}_{2\nu}\right)\Big\}\; T^{\mu'\nu'}_{ab}$$




- WE KNOW (FROM PART 1 OF LECTURE) THAT

$$(\hat{k}_1)_{\mu'}\; T^{\mu'\nu'}_{ab}\; \varepsilon_{\nu'}(2) = 0$$

$$(\hat{k}_2)_{\nu'}\; T^{\mu'\nu'}_{ab}\; \varepsilon_{\mu'}(1) = 0$$

$\Rightarrow$ THIS ELIMINATES ALL <u>TL TERMS</u> !

- FOR LL TERMS $\quad (\mathcal{M} = -i T)$

$$(\hat{k}_1)_{\mu'} \, T_{ab}^{\mu'\nu'} = i g^2 f_{abc} \frac{\lambda_c}{2} \, \bar{v}(P_2, s_2) \frac{\hat{k}_2^{\nu'} \not{k}_1}{2 k_1 \cdot k_2} \, u(P_1, s_1)$$

$$\downarrow \qquad \bar{v}\,(\not{k}_1 + \not{k}_2)\, u = 0$$

$$\boxed{(\hat{k}_1)_{\mu'} \, T_{ab}^{\mu'\nu'} = \hat{k}_2^{\nu'} \cdot \underbrace{i g^2 f_{abc} \frac{\lambda_c}{2} \, \bar{v}(P_2, s_2) \frac{\not{k}_1 - \not{k}_2}{4 k_1 \cdot k_2} \, u(P_1, s_1)}_{\equiv \, T_{ab}}}$$

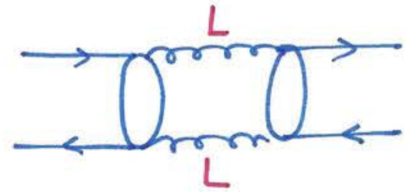
( SYMMETRIC UNDER EXCHANGE OF 2 GLUONS )

ANALOGOUSLY

$$(\hat{k}_2)_{\nu'} \, T_{ab}^{\mu'\nu'} = \hat{k}_1^{\mu'} \, T_{ab}$$

$$\therefore \quad \Delta T = \frac{1}{2} \int d\rho^{(2)} \cdot \frac{1}{4} \, T_{ab}^{+} \Big( \underbrace{(\hat{k}_1 \cdot \hat{k}_2)^2}_{4} + \underbrace{(\hat{k}_1 \cdot \hat{k}_2)^2}_{4} + 0 + 0 \Big) T_{ab}$$

$$\boxed{\Delta T = \int d\rho^{(2)} \, T_{ab}^{+} \, T_{ab}}$$




$\Downarrow$

UNITARITY VIOLATED BY SUM OF DIAGRAMS (A) – (E)

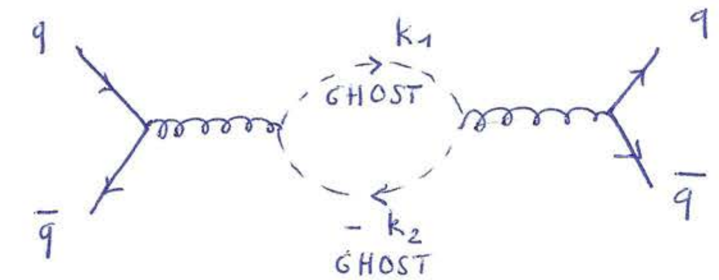
FOR <u>NON</u> – ABELIAN GAUGE THEORY !?

- 'RESTORATION' OF UNITARITY

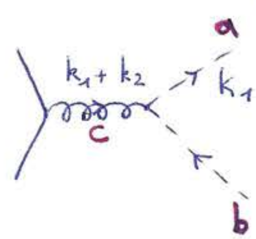
$\Downarrow$

ADD EXTRA UNPHYSICAL FIELDS ('GHOSTS')
SO THAT THEY RESTORE UNITARITY

$\Delta T$ CAN BE INTERPRETED AS GHOST CONTRIBUTION



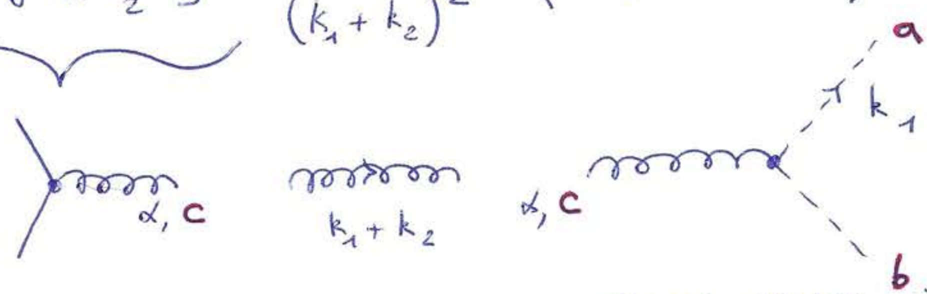

$$\Delta T = -\int d\rho^{(2)} \, (-1) \, T^{\dagger}_{ab} \, \underbrace{T_{ab}}$$




$$T_{ab} = \bar{v}\left[-ig\,\gamma_\alpha \frac{\lambda_c}{2}\right] u \cdot \frac{1}{2\,k_1 \cdot k_2} \cdot \left(-g\, f_{abc}\, k_1^{\alpha}\right)$$

$$= i \; \underbrace{\bar{v}\left[-ig\,\gamma_\alpha \frac{\lambda_c}{2}\right] u} \; \frac{-i}{(k_1+k_2)^2} \left(-g\, f_{abc}\, k_1^{\alpha}\right)$$

TO CONVERT
T-MATRIX
TO S-MATRIX




GLUON- GHOST
VERTEX

GHOSTS APPEAR AS SPINLESS PARTICLES

BUT SATISFY GRASSMANN ALGEBRA, i.e. ANTI-COMMUTE (cf. FERMION)

FACTOR $(-1)$ FOR LOOP

$\downarrow$

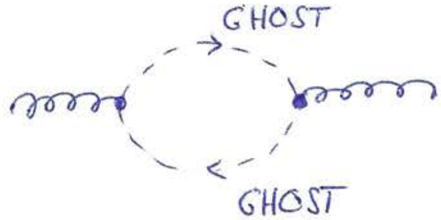
GHOSTS ARE UNPHYSICAL

DO NOT SATISFY SPIN-STATISTICS THEOREM !

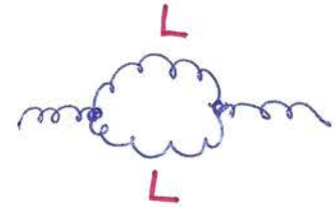
ROLE: SERVE TO CANCEL UNPHYSICAL L POL. DEGREES OF FREEDOM IN GLUON LOOPS

EACH TIME WE HAVE

WE SHOULD ADD




WHICH CANCELS

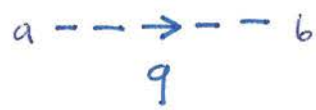



$\Longrightarrow$ $\boxed{\text{UNITARITY IS RESTORED}}$ !

(OR ALTERNATIVELY, USE UNITARY GAUGE ('T HOOFT) SO THAT ONLY T GLUON POL. STATES PROPAGATE)

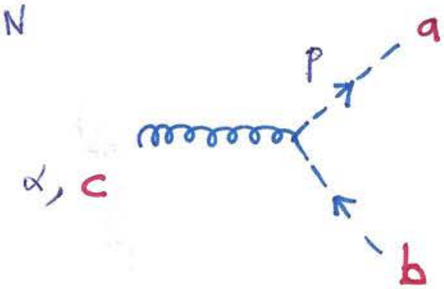
## ⇒ FEYNMAN RULES FOR GHOSTS

- PROPAGATOR




$$\boxed{\frac{i}{q^2}\,\delta_{ab}}$$

- GHOST-GLUON VERTEX




$$\boxed{-\,g\,f_{abc}\;p^{\alpha}}$$

OUTGOING GHOST LINE

- FOR EACH GHOST LOOP ⟹ FACTOR $(-1)$

AS FOR FERMIONS

∴ QUANTIZATION OF QCD AMOUNTS TO ADD TO $\mathcal{L}_{QCD}$

$$\boxed{\begin{aligned} &\mathcal{L}_{QCD} \longrightarrow \mathcal{L}_{QCD} + \mathcal{L}_{GAUGE\text{-}FIXING} + \mathcal{L}_{GHOST} \\ &\mathcal{L}_{GHOST} = \left(\partial_\mu \chi_a\right)\left[\partial^\mu \delta_{ab} + g\,f_{abc}\,A^\mu_c\right]\chi_b \end{aligned}}$$