Theoretical Elementary Particle Physics
Exercise 3

26 November 2018

1 SU(3)$_C$ group (20 points)

Consider again the octet of gluons

$$
\begin{aligned}
g_1 &= r\bar{g}, \quad g_2 = r\bar{b}, \quad g_3 = g\bar{r}, \\
g_4 &= g\bar{b}, \quad g_5 = b\bar{r}, \quad g_6 = b\bar{g}, \\
g_7 &= \frac{1}{\sqrt{2}}(r\bar{r} - g\bar{g}), \\
g_8 &= \frac{1}{\sqrt{6}}(r\bar{r} + g\bar{g} - 2b\bar{b}) \qquad (1)
\end{aligned}
$$

(a) **(10 points)** Show that the octet states $|g_7\rangle = \frac{1}{\sqrt{2}}(r\bar{r} - g\bar{g})$ and $|g_8\rangle = \frac{1}{\sqrt{6}}(r\bar{r} + g\bar{g} - 2b\bar{b})$ go into the linear combinations of one another.

$$|g_7'\rangle = a\,|g_7\rangle + b\,|g_8\rangle \ , \quad |g_8'\rangle = c\,|g_7\rangle + d\,|g_8\rangle \qquad (2)$$

Find the numbers a, b, c, d. Does it work for the other pairs of gluons from the octet?

(b) **(10 points)** Calculate the color factor arising from 1-gluon exchange between a quark and an antiquark in a color octet state $|g_8\rangle$, keeping in mind the normalization factor of the gluon.
Hint: Use the method shown in the seminar.

2 Loop integral (80 points)

Consider the loop diagram for ϕ^4 theory, shown on the Fig. 1, where all particles are scalars and $k = k_1 + k_2$.

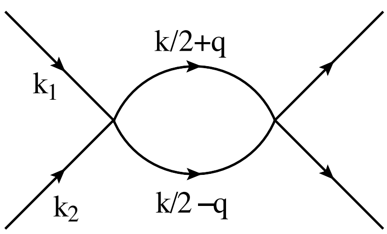

Figure 1: A loop diagram for the scalar theory

(a) **(40 points)** Write down the expression for the amplitude $\mathcal{M}$ corresponding to this diagram and evaluate the loop integral explicitly.

(b) (40 points) Calculate the imaginary part of the amplitude by determining the poles in the complex plane. By comparing your result for the imaginary part with the direct loop diagram calculation in part (a), show that the discontinuity in $\mathcal{M}$ can be obtained by the following replacement of each propagator:

$$\frac{1}{p^2 - m^2 + i\epsilon} \to (-2\pi i)\delta(p^2 - m^2)\theta(p^0) \tag{3}$$