Theoretical Elementary Particle Physics
Exercise 2

12 November 2018

1 Quantization of the scalar field (15 points)

The generating functional for $V = 0$ is defined as $G_0(j) = Z_0(j)/Z_0(0)$. Derive the following result:

$$G_0(j) = \exp\left\{ -\frac{i}{2\hbar} \int d^4x\, d^4y\, j(x)\Delta(x-y)j(y) \right\} \tag{1}$$

2 Path integral for fermions (85 points)

2.1 Grassmann algebra (15 points)

The Grassmann algebra can be implemented using anti-commuting matrices. Derive the matrix representation for the case of $n = 2$ generators. **Hint:** first, show that it is not possible to construct a 2×2 matrix representation.

What is the dimension of matrices for the case of n generators?

2.2 Berezin integral (50 points)

Consider the Gaussian integral over Grassmann variables. It can be shown, that

$$\left(\int \prod_{i=1}^{N} d\bar{\theta}_i d\theta_i \right) \exp\left(-\bar{\theta}_i A_{ij} \theta_j\right) = \det A \tag{2}$$

where A_{ij} is $N \times N$ Hermitian matrix. This result was obtained on the lecture by expanding the exponential function in a power series.

(a) **(10 points)** Perform the expansion of $\exp\left(-\bar{\theta}_i A_{ij} \theta_j\right)$ explicitly for $N = 2$.

(b) **(20 points)** Derive (2) by performing a change of variables which diagonalizes A.

(c) **(20 points)** Show that

$$\left(\int \prod_{i=1}^{N} d\bar{\theta}_i d\theta_i \right) \theta_k \bar{\theta}_l \exp\left(-\bar{\theta}_i A_{ij} \theta_j\right) = (\det A) \times (A^{-1})_{kl} \tag{3}$$

2.3 Fermionic expectation value (20 points)

Show for the fermionic expectation value of two point function that

$$\langle \theta_i \bar{\theta}_j \rangle = (A^{-1})_{ij} \tag{4}$$