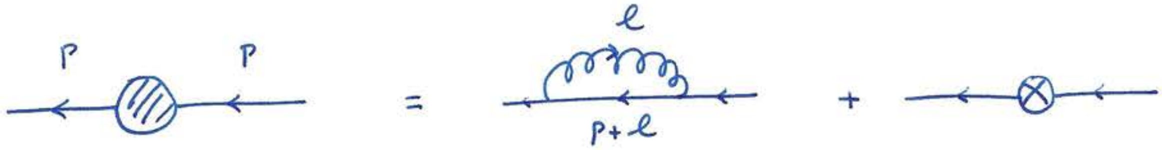
QUARK SELF-ENERGY



$$-i\Sigma_Q(p) = -i\Sigma_{Q,\ell}(p) - i\Sigma_{Q,ct}(p)$$

$$\Rightarrow -i\Sigma_{Q,\ell}(p) = \int\frac{d^D\ell}{(2\pi)^D}\,\frac{i}{\ell^2}\left[-g_{\mu\nu} + (1-\xi)\,\ell_\mu\ell_\nu/\ell^2\right]$$

$$\underbrace{\left(\frac{\lambda_a}{2}\frac{\lambda_a}{2}\right)}_{\frac{4}{3}\mathbb{1}}\left[-ig\gamma^\mu\right]\frac{i(\not p+\not\ell+m)}{(p+\ell)^2-m^2}\left[-ig\gamma^\nu\right]$$

$$= \frac{4}{3}g^2\int\frac{d^D\ell}{(2\pi)^D}\,\frac{1}{\ell^2\left[(p+\ell)^2-m^2\right]}\left\{-\gamma_\mu(\not p+\not\ell+m)\gamma^\mu + \frac{(1-\xi)}{\ell^2}\not\ell(\not p+\not\ell+m)\not\ell\right\}$$

$$= \frac{4}{3}g^2\int\frac{d^D\ell}{(2\pi)^D}\,\frac{1}{\ell^2\left[(p+\ell)^2-m^2\right]}\left\{-(2-D)(\not p+\not\ell) - Dm + (1-\xi)(-\not p+m) + (1-\xi)\frac{2p.\ell+\ell^2}{\ell^2}\not\ell\right\}$$

$$= \frac{4}{3}g^2\int_0^1 dx\int\frac{d^D\ell}{(2\pi)^D}\,\frac{1}{\left[(\ell+px)^2+p^2x(1-x)-m^2x\right]^2}$$

$$\cdot\left\{-(2-D)(\not p+\not\ell) - Dm + (1-\xi)(-\not p+m) + (1-\xi)\not\ell\left(1+\frac{2p.\ell}{\ }\right)\right\}$$

Q $\sqrt{2}$

$$= \frac{4}{3} g^2 \int_0^1 dx \left\{ \int \frac{d^D l}{(2\pi)^D} \frac{1}{\left[l^2 + p^2 x(1-x) - m^2 x\right]^2} \right.$$

$$\cdot \left\{ -(2-D)\left(\not p(1-x) + \cancel{\not l}\right) - Dm \right.$$

$$\left. + (1-\xi)(-\not p + m) + (1-\xi)(\cancel{\not l} - \not p x) \right\}$$

$$\left. + 2(1-x) \int \frac{d^D l}{(2\pi)^D} \frac{(1-\xi)}{\left[l^2 + p^2 x(1-x) - m^2 x\right]^3} \underbrace{\left[2p\cdot(l-px)(\not l - \not p x)\right]}_{2p\cdot l \not l \, + \, 2p^2x^2 \not p \, + \, \cancel{\dots}} \right\}$$

$$= \frac{4}{3} g^2 \frac{i}{(4\pi)^2} \frac{1}{\varepsilon} \left\{ \int_0^1 dx \left[2\not p(1-x) - 4m + (1-\xi)(-\not p(1+x) + m) \right.\right.$$

$$\left.\left. + \frac{1}{2}(1-x)(2\not p)(1-\xi) \right] \right\} + \text{FINITE TERMS}$$

$$= \frac{4}{3} g^2 \frac{i}{(4\pi)^2} \frac{1}{\varepsilon} \left\{ \not p \xi - m(3+\xi) \right\} + \text{FINITE TERMS.}$$

$$\boxed{-i\Sigma_{Q,l}(p) = \frac{4}{3} i \frac{g^2}{(4\pi)^2} \frac{1}{\varepsilon} \left\{ \not p \xi - m(3+\xi) \right\} + \text{FINITE TERMS}}$$

$$\Rightarrow -i\Sigma_{Q,ct}(p) = i\left[(Z_2 - 1)\not p - (Z_2 Z_m - 1)m\right]$$

$\Downarrow$

DIVERGENT TERM OF $\Sigma_{Q,\ell}$ IS CANCELLED BY COUNTERTERM

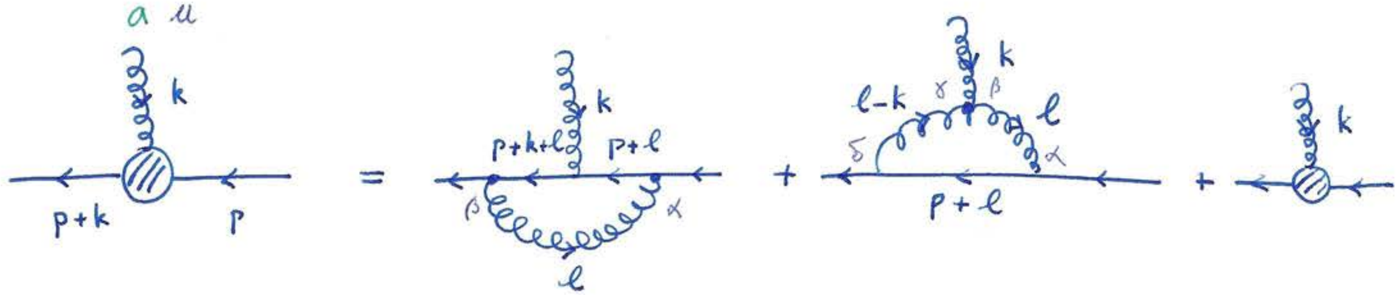
$$\boxed{Z_2 = 1 - \frac{g^2}{(4\pi)^2}\frac{1}{\varepsilon}\frac{4}{3}\xi + O(g^4)}$$

$$Z_2 Z_m = 1 - \frac{g^2}{(4\pi)^2}\frac{1}{\varepsilon}\frac{4}{3}(3+\xi) + O(g^4)$$

$$Z_m = \left[1 - \frac{g^2}{(4\pi)^2}\frac{1}{\varepsilon}\frac{4}{3}(3+\xi) + O(g^4)\right]\left[1 + \frac{g^2}{(4\pi)^2}\frac{1}{\varepsilon}\frac{4}{3}\xi + O(g^4)\right]$$

$$\boxed{Z_m = 1 - \frac{g^2}{(4\pi)^2}\frac{1}{\varepsilon}\,3\cdot\left(\frac{4}{3}\right) + O(g^4)}$$

QUARK - GLUON VERTEX



$$\Lambda^{\mu}_{a\,Q\text{-}GL}(p+k,p,k) = \Lambda^{\mu(1)}_{a,Q\text{-}GL} + \Lambda^{\mu(2)}_{a,Q\text{-}GL} + \Lambda^{\mu(3)}_{a,Q\text{-}GL}$$

$$\Rightarrow \Lambda^{\mu(1)}_{a,Q\text{-}GL}(p+k,p,k)$$

$$= \int \frac{d^D l}{(2\pi)^D} \left[-ig\frac{\lambda_b}{2}\gamma^\beta\right] \frac{i\left[(\not p+\not k+\not l)+m\right]}{(p+k+l)^2-m^2} \left[-ig\frac{\lambda_a}{2}\gamma^\mu\right]$$

$$\cdot \frac{i\left[\not p+\not l+m\right]}{(p+l)^2-m^2} \left[-ig\frac{\lambda_b}{2}\gamma^\alpha\right] \frac{i}{l^2}\left[-g_{\alpha\beta}+(1-\xi)\frac{l_\alpha l_\beta}{l^2}\right]$$

$$= g^3\,\frac{\lambda_b}{2}\frac{\lambda_a}{2}\frac{\lambda_b}{2}$$

$$\int\frac{d^D l}{(2\pi)^D}\,\frac{\gamma^\beta\left[\not p+\not k+\not l+m\right]\gamma^\mu\left[\not p+\not l+m\right]\gamma^\alpha}{\left[(p+k+l)^2-m^2\right]\left[(p+l)^2-m^2\right]l^2}\left[-g_{\alpha\beta}+(1-\xi)\frac{l_\alpha l_\beta}{l^2}\right]$$

$\Downarrow$

TO CALCULATE ONLY DIVERGENT PART

$$= g^3 \underbrace{\frac{\lambda_b}{2}\frac{\lambda_a}{2}\frac{\lambda_b}{2}}_{\longrightarrow -\frac{1}{6}\left(\frac{\lambda_a}{2}\right)}$$

$$\int \frac{d^D\ell}{(2\pi)^D} \frac{-\gamma^\alpha \not\ell \gamma^\mu \not\ell \gamma_\alpha + (1-\xi)\gamma^\mu \ell^2}{\left[(p+k+\ell)^2-m^2\right]\left[(p+\ell)^2-m^2\right]\ell^2} \quad + \text{ FINITE TERMS}$$

$$= -\frac{1}{6}\left(\frac{\lambda_a}{2}\right) g^3 \int \frac{d^D\ell}{(2\pi)^D} \frac{2\not\ell\gamma^\mu\not\ell + (1-\xi)\ell^2\gamma^\mu}{\left[(p+k+\ell)^2-m^2\right]\left[(p+\ell)^2-m^2\right]\ell^2}$$

+ FINITE TERMS

$$= -\frac{1}{6}\left(\frac{\lambda_a}{2}\right) g^3 \; 2\int_0^1 dx\, x \int_0^1 dy \int \frac{d^D\ell}{(2\pi)^D} \frac{2\not\ell\gamma^\mu\not\ell + (1-\xi)\ell^2\gamma^\mu}{\left[\ell^2 + \dots\right]^3} \quad + \text{ FINITE TERMS}$$

$$= -\frac{1}{6}\left(\frac{\lambda_a}{2}\right) g^3 \frac{i}{(4\pi)^2} \frac{1}{\varepsilon} \frac{1}{4} \left\{ + 1\,\gamma^\alpha\gamma^\mu\gamma_\alpha + (1-\xi)\, 4\gamma^\mu \right\}$$

+ FINITE TERMS

$$\boxed{\Lambda^{\mu\,(1)}_{a,\,Q-GL}(p+k,p,k) = -ig\frac{\lambda_a}{2}\gamma^\mu\left[\frac{1}{\varepsilon}\frac{g^2}{(4\pi)^2}\left(-\frac{\xi}{6}\right)\right] \quad + \text{ FINITE TERMS}}$$

$$\frac{\lambda_b}{2}\frac{\lambda_a}{2}\frac{\lambda_b}{2}$$

$$= i f_{bac} \frac{\lambda_c}{2}\frac{\lambda_b}{2} + \frac{\lambda_a}{2}\underbrace{\frac{\lambda_b}{2}\frac{\lambda_b}{2}}_{\frac{4}{3}\mathbb{1}}$$

$$\Rightarrow f_{bac}\frac{\lambda_c}{2}\frac{\lambda_b}{2} = f_{bac}\frac{1}{2}\left(\frac{1}{3}\delta_{bc} + (d_{cbd} + i f_{cbd})\frac{\lambda_d}{2}\right)$$

$$= \frac{i}{2}\underbrace{f_{bac} f_{cbd}}_{3\delta_{ad}}\frac{\lambda_d}{2}$$

$$= \frac{3}{2} i \frac{\lambda_a}{2}$$

$$\therefore \quad \frac{\lambda_b}{2}\frac{\lambda_a}{2}\frac{\lambda_b}{2} = -\frac{3}{2}\frac{\lambda_a}{2} + \frac{4}{3}\frac{\lambda_a}{2}$$

$$\frac{\lambda_b}{2}\frac{\lambda_a}{2}\frac{\lambda_b}{2} = -\frac{1}{6}\frac{\lambda_a}{2}$$

$$\Rightarrow \Lambda^{\mu(2)}_{a,\,Q-GL}(p+k,p,k)$$

$$= \int \frac{d^D l}{(2\pi)^D} \frac{i}{(l-k)^2} \left[-g_{\gamma\delta} + (1-\xi)(l-k)_\gamma (l-k)_\delta / (l-k)^2\right]$$

$$\cdot \frac{i}{l^2} \left[-g_{\beta\alpha} + (1-\xi) l_\alpha l_\beta / l^2\right]$$

$$\cdot \left[-g f_{bac} V^{\gamma\mu\beta}(l-k, k, -l)\right]$$

$$\cdot \left[-ig \gamma^\delta \frac{\lambda_b}{2}\right] i \frac{(\not p + \not l) + m}{(p+l)^2 - m^2} \left[-ig \gamma^\alpha \frac{\lambda_c}{2}\right]$$

$$= +i g^3 f_{abc} \frac{\lambda_b}{2} \frac{\lambda_c}{2}$$

$$\int \frac{d^D l}{(2\pi)^D} \frac{1}{(l-k)^2 l^2 \left[(p+l)^2 - m^2\right]}$$

$$\cdot \left[-g_{\gamma\delta} + (1-\xi)(l-k)_\gamma (l-k)_\delta / (l-k)^2\right]$$

$$\cdot \left[-g_{\alpha\beta} + (1-\xi) l_\alpha l_\beta / l^2\right] V^{\gamma\mu\beta}(l-k, k, -l)$$

$$\cdot \gamma^\delta \left[(\not p + \not l) + m\right] \gamma^\alpha$$

$\Downarrow$

IF ONE IS ONLY INTERESTED IN THE DIVERGENT PART $\sim \frac{1}{\varepsilon}$

$$\Lambda^{\mu(2)}_{a,\, Q-GL}(p+k, p, k)$$

$$= i g^3 f_{abc} \frac{\lambda_b}{2} \frac{\lambda_c}{2}$$

$$\int \frac{d^D \ell}{(2\pi)^D} \frac{1}{(\ell-k)^2 \ell^2 \left[(p+\ell)^2 - m^2\right]}$$

$$\cdot \Big\{ 2\gamma^\mu \ell^2 + 4 \ell^\mu \not{\ell}$$

$$- \frac{(1-\xi)}{(\ell-k)^2} \ell^2 \left[\ell^2 \gamma^\mu - \ell^\mu \not{\ell}\right]$$

$$- \frac{(1-\xi)}{\ell^2} \ell^2 \left[\ell^2 \gamma^\mu - \ell^\mu \not{\ell}\right] \Big\} + \text{FINITE TERMS}$$

$$= i g^3 f_{abc} \frac{\lambda_b}{2} \frac{\lambda_c}{2}$$

$$\cdot \Big\{ 2 \int_0^1 dx \; x \int_0^1 dy \; \frac{i}{(4\pi)^2} \frac{1}{4} \frac{1}{\varepsilon} \left[2\gamma^\mu \cdot 4 + 4\gamma^\mu \right]$$

$$- (1-\xi) \; 2 \int_0^1 dx \, (1-x) \frac{i}{(4\pi)^2} \frac{1}{4} \frac{1}{\varepsilon} \left[4\gamma^\mu - \gamma^\mu \right] \times 2 \Big\}$$

$+$ FINITE TERMS

$$\hookrightarrow \frac{i}{2} f_{bcd}\frac{\lambda_d}{2} + \frac{1}{2} d_{bcd}\frac{\lambda_d}{2} + \frac{1}{6}\delta_{bc}$$

$$= i g^3 \left(f_{abc} \frac{\lambda_b}{2}\frac{\lambda_c}{2} \right) = i\frac{3}{2}\frac{\lambda_a}{2}$$

$$\cdot \frac{i}{(4\pi)^2}\frac{1}{\varepsilon} \underbrace{\left\{ 3\gamma^\mu - (1-\xi)\frac{3}{2}\gamma^\mu \right\}}_{\frac{3}{2}\gamma^\mu(1+\xi)} + \text{FINITE TERMS}$$

$$\boxed{\Lambda^{\mu(2)}_{a,Q-GL}(p+k,p,k) = -ig\frac{\lambda_a}{2}\gamma^\mu\left[\frac{1}{\varepsilon}\frac{g^2}{(4\pi)^2}\frac{9}{4}(1+\xi)\right] + \text{FINITE TERMS}}$$

$$\Rightarrow \boxed{\Lambda^{\mu(3)}_{a,Q-GL}(p+k,p,k) = -ig\frac{\lambda_a}{2}\gamma^\mu\left(Z_{1F}-1\right)}$$

$$\therefore \boxed{Z_{1F} = 1 - \frac{g^2}{(4\pi)^2}\frac{1}{\varepsilon}\left[\frac{9}{4} + \frac{25}{12}\xi\right] + O(g^4)}$$