1	2	3	4	5	6	$\sum$

Name:

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Deadline: Nov 8, 1 pm, 05-124

Symmetries in Physics (WS 2018/2019)
Exercise 1

1. (a) [**1**] Verify that the existence of the unit element implies its uniqueness.
   (b) [**1**] Verify the uniqueness of the inverse element.
   (c) [**1**] Let a^{-1} and b^{-1} be the inverse elements of a and b, respectively. Using the group axioms, determine $(ab)^{-1}$ in terms of a^{-1} and b^{-1}.

2. (a) [**4**] Construct the group multiplication tables for groups of order 4.
   Hints: Each row and each column contains a given group element exactly once. Two group multiplication tables represent the same abstract group, i.e., they express the same group structure, if they coincide after appropriately rearranging rows and columns and renaming the group elements. At the end you should find two independent group structures.
   (b) [**1**] Argue why groups of order 4 are Abelian.

3. [**6**] Work out the group multiplication table for the dihedral group $D_3 = \langle c, b \rangle$ using the defining relations $c^3 = b^2 = (bc)^2 = e$:

e	c	c^2	b	bc	bc^2
c					
c^2					
b					
bc					
bc^2					

Hint: Make use of the defining relations as in

$$cb = ecbe = b^2cbc^3 = b(bc)^2c^2 = bec^2 = bc^2 \quad \text{etc.}$$

4. [**1**] Identify the non-trivial subgroups of D_3.

5. [**3**] Consider the set of matrices

$$\left\{ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} \omega & 0 \\ 0 & \omega^2 \end{pmatrix}, \begin{pmatrix} \omega^2 & 0 \\ 0 & \omega \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & \omega^2 \\ \omega & 0 \end{pmatrix}, \begin{pmatrix} 0 & \omega \\ \omega^2 & 0 \end{pmatrix} \right\},$$

where $\omega^3 = 1$ and $\omega \neq 1$. Using matrix multiplication, show that this set is a realization of D_3 in problem 3. Work out a one-to-one correspondence between the above matrices and the elements of D_3.

6. **[2]** Consider the permutations $P_i \in S_3$:

$$P_1 = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix} \quad P_2 = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}, \quad P_3 = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix},$$
$$P_4 = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}, \quad P_5 = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}, \quad P_6 = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}.$$

Verify explicitly the associative law for

$$(P_2P_4)P_6 = P_2(P_4P_6).$$