

NON-ABELIAN GAUGE THEORY

QCD

⇒ ABELIAN GAUGE THEORY, QED

- FERMION (SPIN 1/2)

DIRAC EQ. $(i \gamma^\mu \partial_\mu - m) \psi(x) = 0$

$$\mathcal{L}_0 = \bar{\psi}(x) (i \gamma^\mu \partial_\mu - m) \psi(x)$$

WITH $\partial_\mu = \frac{\partial}{\partial x^\mu} = (\frac{\partial}{\partial t}, \vec{\nabla})$

- LOCAL PHASE TRANSFORMATION



$\Delta\theta = \theta_2 - \theta_1$
PHASE DIFFERENCE

$$\psi(x) \rightarrow e^{i\theta(x)} \psi(x)$$

$$\partial_\mu \psi(x) \rightarrow e^{i\theta(x)} [\partial_\mu \psi + i(\partial_\mu \theta) \psi]$$

$$\bar{\psi} \psi \rightarrow \bar{\psi} \psi$$

$$\bar{\psi} \gamma^\mu (\partial_\mu \psi) \rightarrow \bar{\psi} \gamma^\mu (\partial_\mu \psi) + i(\partial_\mu \theta) \bar{\psi} \gamma^\mu \psi$$

$$L_0 \rightarrow L_0 - \underbrace{(\partial_\mu \theta) \bar{\Psi} \gamma^\mu \Psi}_{\text{VECTOR}}$$

• INVARIANCE UNDER LOCAL PHASE TRANSFORMATION

INTRODUCE VECTOR FIELD (GAUGE FIELD)

$$\partial_\mu \Rightarrow D_\mu \equiv \partial_\mu + ie A_\mu$$

↑
COVARIANT DERIVATIVE

↑
COUPLING STRENGTH
OF FERMION TO
GAUGE FIELD
(ELECTRIC CHARGE)

$$A_\mu \rightarrow A_\mu - \frac{1}{e} \partial_\mu \theta$$

$$D_\mu \Psi(x) \rightarrow e^{i\theta(x)} \left[\partial_\mu \Psi + i(\partial_\mu \theta) \Psi + ie A'_\mu \Psi \right]$$

$$\Downarrow \quad A'_\mu = A_\mu - \frac{1}{e} \partial_\mu \theta$$

$$= e^{i\theta(x)} \left[\partial_\mu \Psi + ie A_\mu \Psi \right]$$

$$= e^{i\theta(x)} D_\mu \Psi(x)$$

$$\bar{\Psi}(x) \gamma^\mu D_\mu \Psi(x) \rightarrow \bar{\Psi}(x) \gamma^\mu D_\mu \Psi \quad \text{INVARIANT!} \quad \nabla$$

TO GET THEORY WHICH HAS SYMMETRY
UNDER LOCAL PHASE TRANSFORMATION

REPLACE $\partial_\mu \psi \Rightarrow D_\mu \psi$

\Downarrow

REPLACE $\mathcal{L}_0 \Rightarrow \bar{\psi}(x) (i \gamma^\mu D_\mu - m) \psi(x)$

$$= \underbrace{\bar{\psi}(x) (i \gamma^\mu \partial_\mu - m) \psi(x)}_{\mathcal{L}_0} - \underbrace{e \bar{\psi} \gamma^\mu \psi A_\mu}_{\mathcal{L}_{INT}}$$

$\mathcal{L}_0 + \mathcal{L}_{INT}$ IS INVARIANT UNDER LOCAL PHASE TF.

$U(1)$ GAUGE TF

$\mathcal{L}_0 + \mathcal{L}_{INT}$ HAS $U(1)$ GAUGE SYMMETRY

• COUPLING TO PHOTON (GAUGE FIELD)

\hookrightarrow DESCRIBED BY $\mathcal{L}_{INT} = -e \bar{\psi} \gamma^\mu \psi A_\mu$ \hookrightarrow VECTOR POTENTIAL



FEYNMAN RULE

$$\boxed{-ie \gamma^\mu}$$

• GEOMETRIC INTERPRETATION OF COVARIANT DERIVATIVE

→ INFINITESIMAL DISTANCE dx^μ BETWEEN 2 POINTS

$$dx^\mu \cdot \partial_\mu \psi(x) = \psi(x+dx) - \psi(x)$$

$$\rightarrow dx^\mu \cdot D_\mu \psi(x) = dx^\mu \left(\partial_\mu \psi(x) + ie A_\mu(x) \psi(x) \right)$$



TAKE AS GAUGE FIELD

$$A_\mu = -\frac{1}{e} \partial_\mu \theta$$

$$= dx^\mu \partial_\mu \psi(x) - i dx^\mu (\partial_\mu \theta) \psi(x)$$

$$= \psi(x+dx) - \psi(x) - i [\theta(x+dx) - \theta(x)] \psi(x)$$

$$= \psi(x+dx) - \left[1 + i (\theta(x+dx) - \theta(x)) \right] \psi(x)$$

$$\approx \psi(x+dx) - \exp \{ i \theta(x+dx) - i \theta(x) \} \psi(x)$$



FACTOR WHICH COMPENSATES
PHASE DIFFERENCE IN FIELD $\psi(x)$
AT POINTS x AND $x+dx$

$$\Phi(x+dx, x)$$

$$\begin{aligned} \rightarrow \quad \underline{\Phi}(x+dx, x) &\equiv \exp \left\{ i dx^\mu \partial_\mu \theta \right\} \\ &= \exp \left\{ -ie dx^\mu A_\mu(x) \right\} \end{aligned}$$

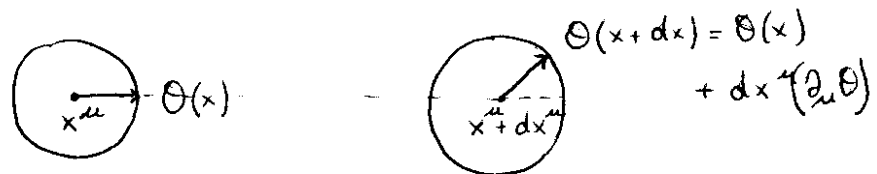
" PARALLEL TRANSPORT "

‡ COMPARE FIELDS COMPENSATED FOR THEIR PHASE DIFFERENCE
AT 2 POINTS
INSTEAD OF FIELDS THEMSELVES

$$dx^\mu \mathbb{D}_\mu \psi(x) = \psi(x+dx) - \underline{\Phi}(x+dx, x) \psi(x)$$

$$-e A_\mu(x) = \partial_\mu \theta$$

↳ GAUGE FIELD → INTERPRETATION AS
RELATIVE ORIENTATION (IN DIRECTION μ)
OF PHASE AT 2 NEIGHBOURING POINTS



→ FOR A FINITE PATH ρ : $x \xrightarrow{\rho} y$

$$\underline{\Phi}_\rho(y, x) \equiv \exp \left\{ -ie \int_\rho dx^\mu A_\mu(x) \right\}$$

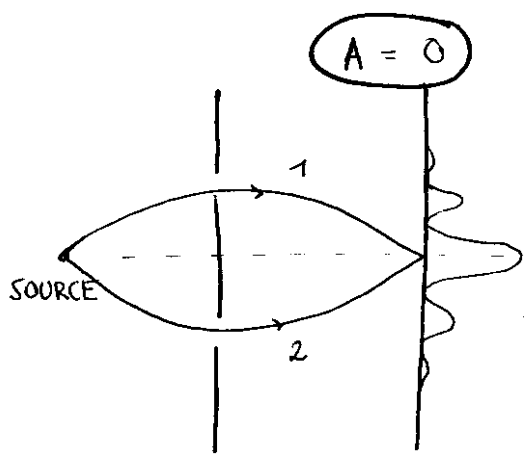
$\bar{\psi}(y) \underline{\Phi}_\rho(y, x) \psi(x)$ IS GAUGE INVARIANT ($U(1)$)

CAN ONE SHOW EXPERIMENTALLY THAT A GAUGE FIELD (GAUGE) IS EQUIVALENT TO A PHASE DIFFERENCE FOR FIELD ψ ? 6

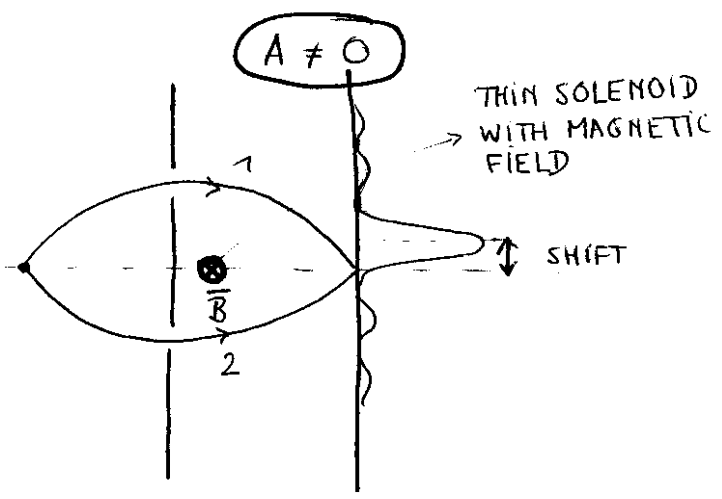
YES !

AHARONOV - BOHM EXPERIMENT

2 SLIT EXPERIMENT



NO PHASE DIFFERENCE
FOR e^- ALONG 1 OR 2



PHASE DIFFERENCE
FOR e^- ALONG 1 OR 2

$$\Delta\theta = \left(-e \int_{P_2} dx^\mu A_\mu \right) - \left(-e \int_{P_1} dx^\mu A_\mu \right)$$

$$= -e \oint dx^\mu A_\mu$$

STOKES LAW \downarrow $C \rightarrow$ CLOSED CONTOUR (CCW)

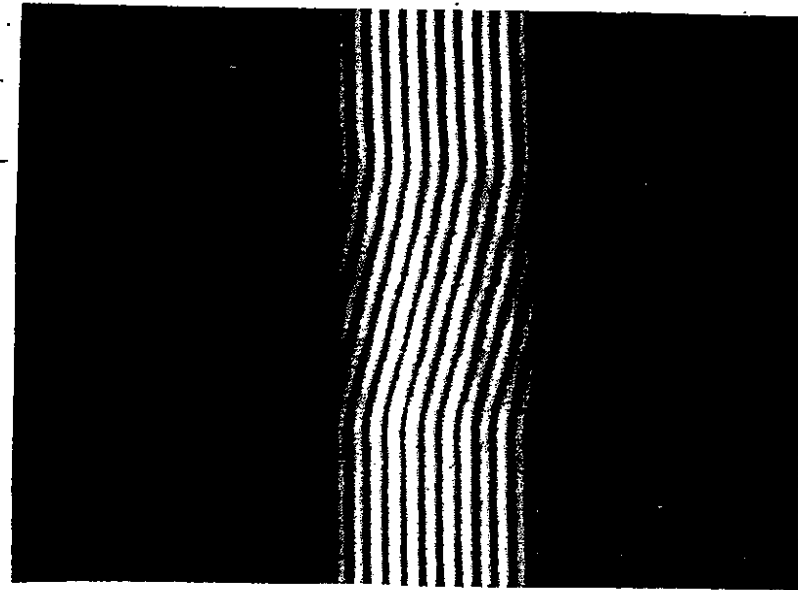
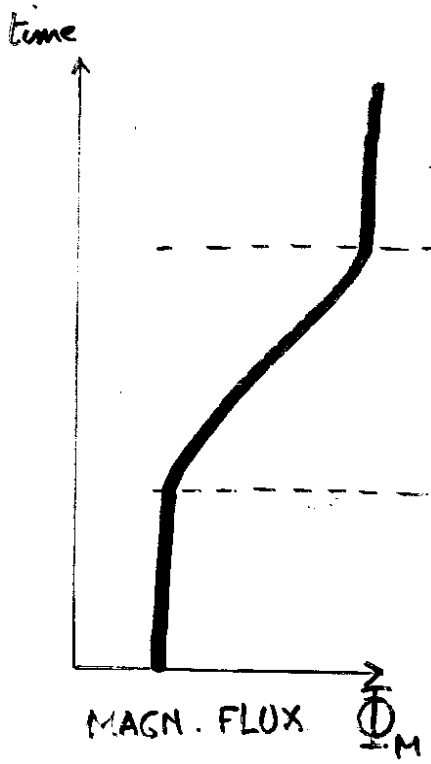
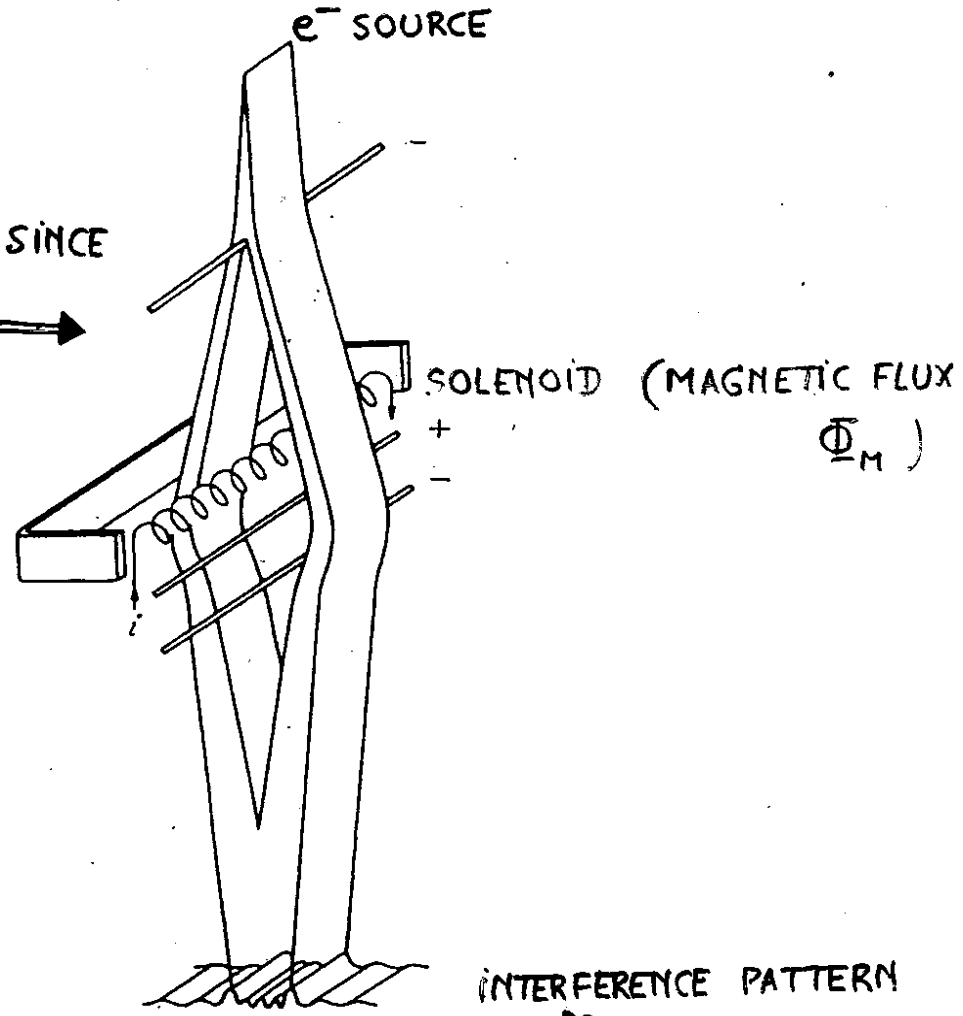
$$= -e \Phi_M$$

\hookrightarrow MAGNETIC FLUX THROUGH LOOP

\Downarrow
INTERFERENCE PATTERN CHANGED !

EXPERIMENTAL OBSERVATION OF AHARONOV-BOHM EFFECT

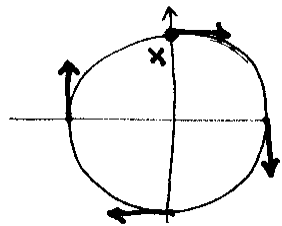
- FIRST EXPERIMENT: CHAMBERS (1960)
- SEVERAL TECHNIQUES SINCE e.g. BAYH (1962) →



IN PRESENCE OF GAUGE FIELD A ,
 BY GOING AROUND A CLOSED CURVE ,
 THE PHASE OF FERMION FIELD HAS CHANGED
 BY AMOUNT $-e\Phi_M$

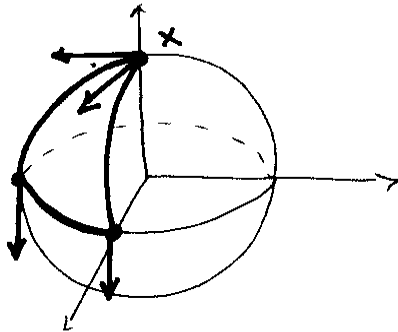
GEOMETRIC ANALOGY

→ PERSON LIVING IN FLAT 2 DIM WORLD
 GOING AROUND CLOSED CURVE, HOLDING AN ARROW



$$\Delta\theta = 0$$

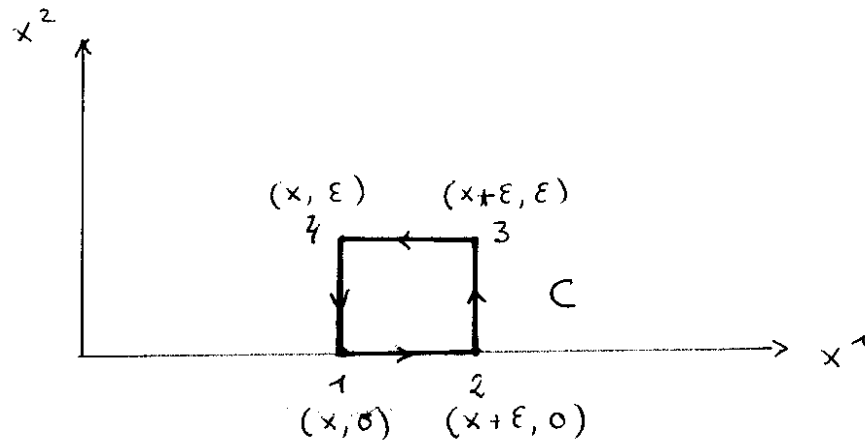
→ PERSON LIVING IN CURVED 2 DIM WORLD (SURFACE OF SPHERE)
 GOING AROUND CLOSED CURVE



$$\Delta\theta = 90^\circ$$

• EM FIELD TENSOR

PARALLEL TRANSPORT ALONG INFINITESIMAL CLOSED CURVE C



$$\bar{\Phi}_C(x) = \exp \left\{ -ie \oint_C dx^\mu A_\mu \right\}$$

$$= \underline{\Phi}(1,4) \underline{\Phi}(4,3) \underline{\Phi}(3,2) \underline{\Phi}(2,1)$$

$$\underline{\Phi}(2,1) = \exp \left\{ -ie \int_1^2 dx^\mu A_\mu \right\}$$

$$= \exp \left\{ +ie \epsilon A^1 \left(x + \frac{\epsilon}{2}, 0 \right) + O(\epsilon^3) \right\}$$

$$\underline{\Phi}(4,3) = \exp \left\{ -ie \epsilon A^1 \left(x + \frac{\epsilon}{2}, \epsilon \right) + O(\epsilon^3) \right\}$$

$$\underline{\Phi}(4,3) \underline{\Phi}(2,1) = \exp \left\{ -ie \epsilon \left[A^1 \left(x + \frac{\epsilon}{2}, \epsilon \right) - A^1 \left(x + \frac{\epsilon}{2}, 0 \right) \right] + O(\epsilon^3) \right\}$$

$$= \exp \left\{ -ie \epsilon^2 \frac{\partial}{\partial x^2} A^1(x, 0) + O(\epsilon^3) \right\}$$

ANALOGOUSLY

$$\underline{\Phi}(1,4) \underline{\Phi}(3,2) = \exp \left\{ +ie \epsilon^2 \frac{\partial}{\partial x^1} A^2(x, 0) + O(\epsilon^3) \right\}$$

$$\begin{aligned} \Phi_c(x) &= \exp \left\{ ie \epsilon^2 \left[\frac{\partial}{\partial x^1} A^2 - \frac{\partial}{\partial x^2} A^1 \right] + O(\epsilon^3) \right\} \\ &= \exp \left\{ -ie \epsilon^2 \underbrace{\left(\partial^1 A^2 - \partial^2 A^1 \right)}_{\substack{\text{SURFACE} \\ \downarrow \\ F^{12}: \text{EM FIELD TENSOR}}} + O(\epsilon^3) \right\} \end{aligned}$$

(cf. MAGN. FLUX IN AHARONOV-BOHM EXP.)

$F^{\mu\nu} \equiv \partial^\mu A^\nu - \partial^\nu A^\mu$

⇒ COMPONENTS GIVE \vec{E}, \vec{B} FIELDS

$$\begin{cases} E^i = -F^{0i} \\ B^i = -\frac{1}{2} \epsilon^{ijk} F^{jk} \end{cases}$$

BECAUSE

$$\Phi(y, x) \xrightarrow{U(1)} \exp\{i\theta(y)\} \Phi(y, x) \exp\{-i\theta(x)\}$$

$$\Phi_c(x) \xrightarrow{U(1)} \Phi_c(x)$$

(x=y) INVARIANT

$$F^{\mu\nu} \xrightarrow{U(1)} F^{\mu\nu}$$

EM FIELD TENSOR IS GAUGE INVARIANT !

→ WE CAN ALSO SHOW THAT

$$\begin{aligned} [D^\mu, D^\nu] \psi &= [\partial^\mu, \partial^\nu] \psi + ie \left([\partial^\mu, A^\nu] - [\partial^\nu, A^\mu] \right) \psi \\ &\quad - e^2 [A^\mu, A^\nu] \psi \\ &= ie (\partial^\mu A^\nu - \partial^\nu A^\mu) \psi \end{aligned}$$

$[D^\mu, D^\nu] = ie F^{\mu\nu}$

⇒ NON-ABELIAN GAUGE THEORY, QCD

• QUARKS

$$\mathcal{L}_{\text{FREE QUARKS}}^{\text{FREE}} = \bar{q} (i \gamma^\mu \partial_\mu - m) q$$

QUARKS COME IN 3 COLORS (r, g, b)

$$q = \begin{pmatrix} q_r \\ q_g \\ q_b \end{pmatrix}$$

SU(3)_c GAUGE SYMMETRY: PHYSICS IS INVARIANT UNDER SU(3) LOCAL 'ROTATIONS' OF 3 QUARK COLORS

$$U \in \text{SU}(3)_c : q \rightarrow U q$$

$$U = \exp \{ i \theta_a(x) t_a \}$$

$$t_a = \frac{\lambda_a}{2} ; a=1 \dots 8$$

(SU(3)_c GENERATORS

WITH

$$[t_a, t_b]_- = i f_{abc} t_c$$

• COVARIANT DERIVATIVE

REPLACE $\partial_\mu \Rightarrow$

$$D_\mu \equiv \partial_\mu + i g \underbrace{t_a A_\mu^a}_{A_\mu}$$

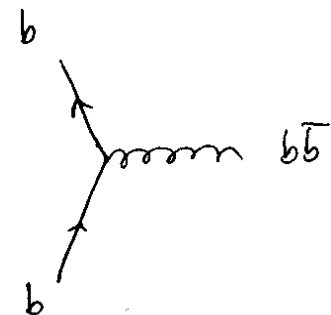
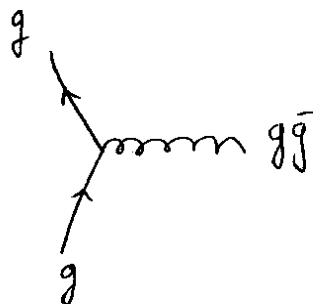
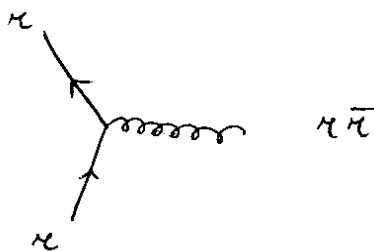
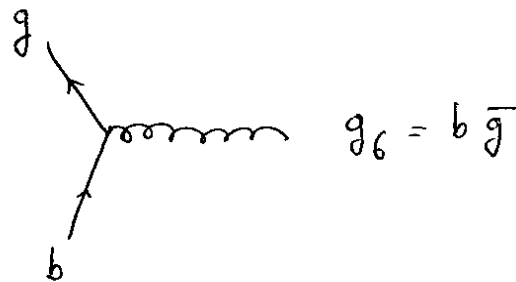
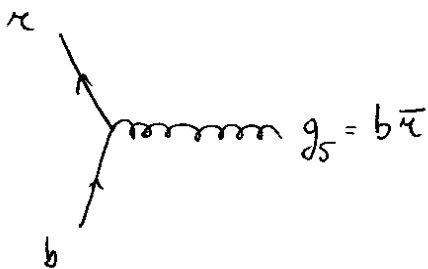
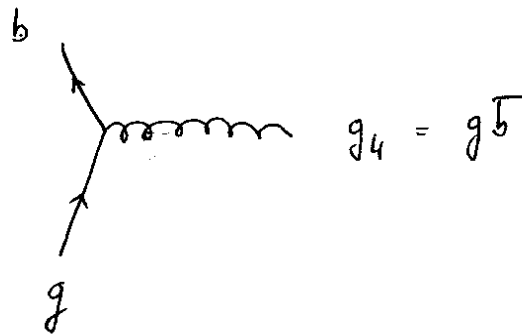
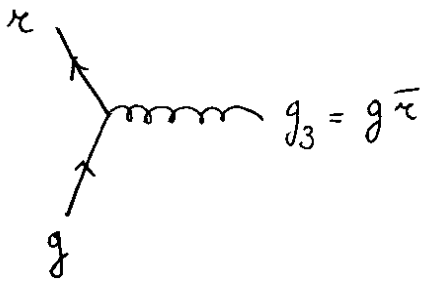
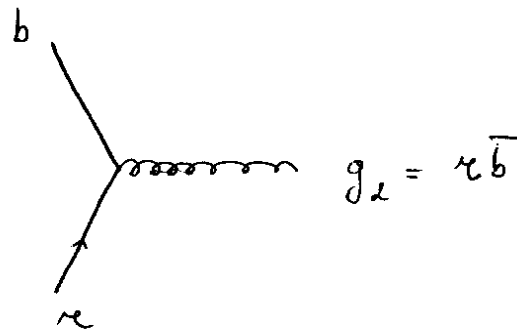
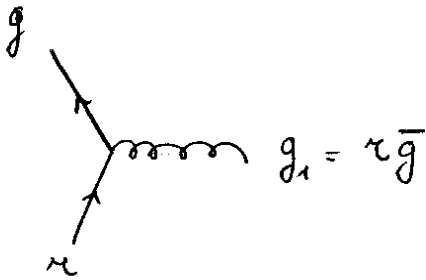
→ 3x3 MATRIX

A_μ^a : GAUGE FIELD → 8 GLUONS
 g : COUPLING OF QUARKS TO GLUONS

8 POSSIBLE TF OF 3 QUARK COLORS



8 GLUONS



ONLY 2 COMBINATIONS CORRESPOND WITH COLORED GLUON

$\frac{1}{\sqrt{3}} (u\bar{u} + g\bar{g} + b\bar{b})$ IS COLOR SINGLET [1]

$$g_7 = \frac{1}{\sqrt{2}} (u\bar{u} - g\bar{g})$$

$$g_8 = \frac{1}{\sqrt{6}} (u\bar{u} + g\bar{g} - 2b\bar{b})$$

WANT $\bar{q} \gamma^\mu \mathcal{D}_\mu q$ TO BE $SU(3)_c$ GAUGE INVARIANT

BECAUSE $\bar{q} \xrightarrow{SU(3)_c} \bar{q} U^\dagger$

$$\mathcal{D}_\mu q \xrightarrow{SU(3)_c} U (\mathcal{D}_\mu q)$$

• HOW DOES A_μ TRANSFORM?

$$\mathcal{D}_\mu q = \partial_\mu q + ig A_\mu q$$

$$\xrightarrow{SU(3)_c} \partial_\mu (Uq) + ig A'_\mu U q$$

$$= (\partial_\mu U) q + U (\partial_\mu q) + ig A'_\mu U q$$

ON THE OTHER HAND

$$\mathcal{D}_\mu q \xrightarrow{SU(3)_c} U (\mathcal{D}_\mu q)$$

$$= U (\partial_\mu q) + ig U A_\mu q$$

$$\circ \circ \quad (\partial_\mu U) + ig A'_\mu U = ig U A_\mu$$

$$A'_\mu U = U A_\mu + \frac{i}{g} (\partial_\mu U)$$

$$\boxed{A_\mu \xrightarrow{SU(3)_c} A'_\mu = U A_\mu U^{-1} + \frac{i}{g} (\partial_\mu U) U^{-1}}$$

↳ WE CAN WRITE THIS EQUIVALENTLY

$$\text{USING } U U^{-1} = \mathbb{1}$$

↓

$$(\partial_\mu U) U^{-1} + U (\partial_\mu U^{-1}) = 0$$

$$A_\mu \rightarrow U A_\mu U^{-1} - \frac{i}{g} U (\partial_\mu U^{-1})$$

↳ TRANSFORMATION LAW FOR INDIVIDUAL FIELDS A_μ^a , $a=1 \dots 8$

$$U = \exp \{ i \theta_a t_a \} \approx \mathbb{1} + i \theta_a t_a + O(\theta^2)$$

INFINIT. TF.

$$A_\mu^a t_a \rightarrow (\mathbb{1} + i \theta_b t_b) A_\mu^c t_c (\mathbb{1} - i \theta_{b'} t_{b'})$$

$$- \frac{i}{g} (\mathbb{1} + i \theta_b t_b) (-i (\partial_\mu \theta_a) t_a)$$

$$\approx A_\mu^a t_a + i \theta_b A_\mu^c [t_b, t_c]$$

$$- \frac{1}{g} (\partial_\mu \theta_a) t_a + O(\theta^2)$$

$$= \left[A_\mu^a - f_{abc} \theta_b A_\mu^c - \frac{1}{g} (\partial_\mu \theta_a) \right] t_a + O(\theta^2)$$

$$\circ \circ \quad A_\mu^a \xrightarrow{SU(3)_c} A_\mu^a - \frac{1}{g} (\partial_\mu \theta_a) - f_{abc} \theta_b A_\mu^c$$

↓
↓
 QED LIKE TERM $SU(3)_c$ TERM !

• EM FIELD TENSOR

$$\rightarrow [D_\mu, D_\nu]_- \equiv ig F_{\mu\nu}$$

$$F_{\mu\nu} \equiv t_a \cdot F_{\mu\nu}^a$$

$$\begin{aligned} ig F_{\mu\nu} &= [\partial_\mu + ig A_\mu, \partial_\nu + ig A_\nu]_- \\ &= ig (\partial_\mu A_\nu - \partial_\nu A_\mu) + (ig)^2 [A_\mu, A_\nu] \end{aligned}$$

$$\Downarrow$$

$$F_{\mu\nu} = \underbrace{\partial_\mu A_\nu - \partial_\nu A_\mu}_{\text{cf. ABELIAN CASE}} + ig \underbrace{[A_\mu, A_\nu]}_{\text{SELF-INTERACTIONS BETWEEN } g \text{ NON-ABELIAN TERM}}$$

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g f_{abc} A_\mu^b A_\nu^c$$

→ TRANSFORMATION LAW

$$\begin{aligned} [D_\mu, D_\nu]_- \psi &\rightarrow U [D_\mu, D_\nu]_- U^\dagger (U\psi) \\ &= ig F_{\mu\nu}' (U\psi) \end{aligned}$$

$$F^{\mu\nu} \rightarrow F'^{\mu\nu} = U F^{\mu\nu} U^\dagger$$

● GAUGE-FIELD TERM IN \mathcal{L}

WE NEED A GAUGE INV. TERM

→ IN CASE OF U(1) $\mathcal{L}_{\text{GAUGE}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$

$$F_{\mu\nu} \rightarrow F_{\mu\nu}$$

→ IN CASE OF SU(3)_c: $F_{\mu\nu} \rightarrow U F_{\mu\nu} U^\dagger$

GAUGE INV. TERM, BILINEAR IN $F_{\mu\nu}$:

$$\text{Tr} \left(F_{\mu\nu} F^{\mu\nu} \right) = F_{\mu\nu}^a F^{b\mu\nu} \underbrace{\text{Tr} (t_a t_b)}_{\frac{1}{2} \delta_{ab}}$$

$$\mathcal{L}_{\text{YM}} = -\frac{1}{2} \text{Tr} \left\{ F_{\mu\nu} F^{\mu\nu} \right\} = -\frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a$$

↑
YANG-MILLS

SUM OVER 8 GLUONS

BECAUSE $F^{\mu\nu}$ HAS $[A^\mu, A^\nu]$ TERM

\mathcal{L}_{YM} HAS CUBIC & QUARTIC TERMS IN A

↳ SELF-INTERACTIONS AMONG GLUONS

FEATURE OF NON-ABELIAN GAUGE THEORY

⇒ FEYNMAN RULES IN QCD

$$\mathcal{L}_{\text{QCD}} = \bar{q} (i \gamma^\mu \partial_\mu - m) q - g \bar{q} \gamma^\mu \frac{\lambda_a}{2} q A_\mu^a - \frac{1}{4} F^{a\mu\nu} F_{\mu\nu}^a$$

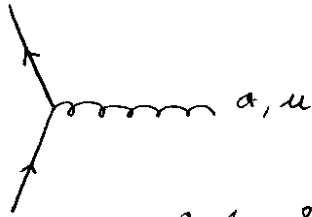
• FEYNMAN RULE FOR $q\bar{q}g$ VERTEX

FEYNMAN RULE



S-MATRIX

"S = iL"



$$a = 1 \dots 8$$

$$\mu = 0, 1, 2, 3$$

$$-ig \gamma^\mu \frac{\lambda_a}{2}$$

• $\mathcal{L}_{\text{YM}} = -\frac{1}{4} F^{a\mu\nu} F_{\mu\nu}^a$

$$= -\frac{1}{4} \left(\partial^\mu A^{a\nu} - \partial^\nu A^{a\mu} - g f_{abc} A^{b\mu} A^{c\nu} \right)$$

$$\cdot \left(\partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g f_{abc} A_\mu^b A_\nu^c \right)$$

$$= -\frac{1}{4} \left(\partial^\mu A^{a\nu} - \partial^\nu A^{a\mu} \right) \left(\partial_\mu A_\nu^a - \partial_\nu A_\mu^a \right) \Rightarrow \text{FREE GLUONS}$$

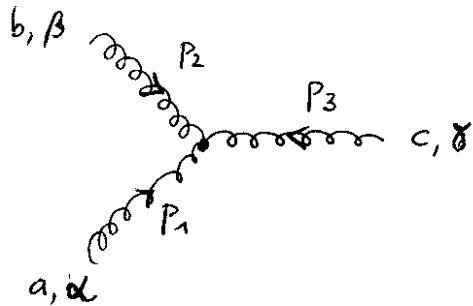
$$+ g f_{abc} \left(\partial^\mu A^{a\nu} \right) A_\mu^b A_\nu^c \Rightarrow 3g \text{ INTERACTION}$$

$$- \frac{1}{4} g^2 f_{abc} f_{ade} A^{b\mu} A^{c\nu} A_\mu^d A_\nu^e \Rightarrow 4g \text{ INTERACTION}$$

NOTE: SAME COUPLING g APPEARS IN $3g$ AND $4g$ INTERACTIONS

• FEYNMAN RULE FOR 3g VERTEX

$$\mathcal{L}_{3g} = g f_{abc} (\partial^\mu A^{a\nu}) A_\mu^b A_\nu^c$$



$$g f_{abc} \left\{ (P_1)^\mu g^{\alpha\nu} \left(\underbrace{g_\mu^\beta g_\nu^\gamma}_{\text{wavy}} - \underbrace{g_\nu^\beta g_\mu^\gamma}_{\text{wavy}} \right) \right. \\ + (P_2)^\mu g^{\beta\nu} \left(\underbrace{g_\mu^\gamma g_\nu^\alpha}_{\text{wavy}} - \underbrace{g_\nu^\gamma g_\mu^\alpha}_{\text{wavy}} \right) \\ \left. + (P_3)^\mu g^{\gamma\nu} \left(\underbrace{g_\mu^\alpha g_\nu^\beta}_{\text{wavy}} - \underbrace{g_\nu^\alpha g_\mu^\beta}_{\text{wavy}} \right) \right\}$$

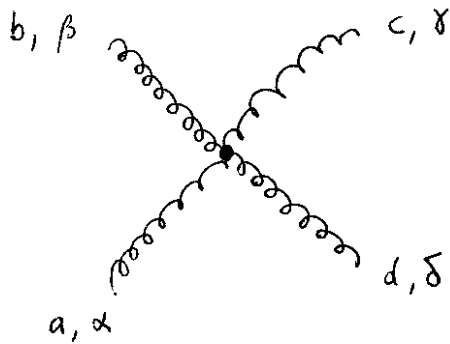
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FEYNMAN RULE

$$\begin{aligned} & - g f_{abc} \left\{ g^{\alpha\beta} (P_1 - P_2)^\gamma \right. \\ & \quad + g^{\beta\gamma} (P_2 - P_3)^\alpha \\ & \quad \left. + g^{\gamma\alpha} (P_3 - P_1)^\beta \right\} \\ & \equiv - g f_{abc} V^{\alpha\beta\gamma} (P_1, P_2, P_3) \end{aligned}$$

• FEYNMAN RULE FOR 4g VERTEX

$$\mathcal{L}_{4g} = -\frac{1}{4} g^2 f_{eab} f_{ecd'} A^{a\mu} A^{b\nu} A_{\mu}^{c'} A_{\nu}^{d'}$$



$$- \textcircled{4} \cdot \frac{i}{4} g^2 \left\{ \underline{f_{eab} f_{ecd}} (g_{\alpha}^{\mu} g_{\beta}^{\nu} g_{\delta\mu} g_{\delta\nu}) \right.$$

4 POSSIBLE WAYS TO
CHOOSE FIRST GLUON

$$+ \underline{f_{eab} f_{edc}} (g_{\alpha}^{\mu} g_{\beta}^{\nu} g_{\delta\mu} g_{\delta\nu})$$

$$+ \underline{f_{eac} f_{ebd}} (g_{\alpha}^{\mu} g_{\gamma}^{\nu} g_{\beta\mu} g_{\delta\nu})$$

$$+ \underline{f_{eac} f_{edb}} (g_{\alpha}^{\mu} g_{\gamma}^{\nu} g_{\delta\mu} g_{\beta\nu})$$

$$+ \underline{f_{ead} f_{ebc}} (g_{\alpha}^{\mu} g_{\delta}^{\nu} g_{\beta\mu} g_{\delta\nu})$$

$$+ \underline{f_{ead} f_{ecb}} (g_{\alpha}^{\mu} g_{\delta}^{\nu} g_{\gamma\mu} g_{\beta\nu}) \left. \right\}$$

↓

$$- i g^2 \left\{ f_{abe} f_{cde} (g_{\alpha\gamma} g_{\beta\delta} - g_{\alpha\delta} g_{\beta\gamma}) \right.$$

$$+ f_{ace} f_{bde} (g_{\alpha\beta} g_{\gamma\delta} - g_{\alpha\delta} g_{\beta\gamma})$$

$$\left. + f_{ade} f_{bce} (g_{\alpha\beta} g_{\gamma\delta} - g_{\alpha\gamma} g_{\beta\delta}) \right\}$$

$$\equiv -i g^2 W_{\alpha\beta\gamma\delta}^{abcd} \rightarrow \text{FULLY SYMMETRIC UNDER INTERCHANGE OF 2 INDICES}$$