

Übungsblatt 9

Theoretische Physik 3 : QM SS2018

Dozent : Prof. M. Vanderhaeghen

20.06.2017

Übung 0.

Wie viel Zeit hat es gebraucht, um die Aufgaben zu erledigen?

Übung 1. (40 Punkte)

In dieser ersten Übung werden wir üben, mit Hilfe von Clebsch-Gordan-Tabellen zwei Drehimpulse j_1 und j_2 zu koppeln. Es sei daran erinnert, dass die gekoppelten Zustände, die durch den Gesamtdrehimpuls J und seine Projektion M charakterisiert sind, über die Vollständigkeitsrelation in der entkoppelten Basis erweitert werden können:

$$|JM\rangle = \sum_{m_1=-j_1}^{j_1} \sum_{m_2=-j_2}^{j_2} |j_1 m_1 j_2 m_2\rangle \langle j_1 m_1 j_2 m_2 | JM\rangle.$$

Die Expansionskoeffizienten, $\langle j_1 m_1 j_2 m_2 | JM\rangle$, sind die Clebsch-Gordan-Koeffizienten, die in der folgenden Tabelle zu finden sind:

34. CLEBSCH-GORDAN COEFFICIENTS. SPHERICAL HARMONICS

Note: A square-root sign is to be understood over every coefficient, e.g., for $-8/15$ read $-\sqrt{8/15}$.

Notation:

J	J	...
M	M	...

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$Y_\ell^{-m} = (-1)^m Y_\ell^{m*}$	$a_{m,0}^\ell = \sqrt{\frac{4\pi}{2\ell+1}} Y_\ell^m e^{-im\phi}$	<table border="1" style="margin-left: 20px;"> <tr><td>$\langle j_1 j_2 m_1 m_2 j_1 j_2 JM\rangle$</td></tr> <tr><td>$= (-1)^{J-j_1-j_2} \langle j_2 j_1 m_2 m_1 j_2 j_1 JM\rangle$</td></tr> </table>		$\langle j_1 j_2 m_1 m_2 j_1 j_2 JM\rangle$	$= (-1)^{J-j_1-j_2} \langle j_2 j_1 m_2 m_1 j_2 j_1 JM\rangle$																																																																																								
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- a) (25 p.) Notiere alle mögliche Zustände $|JM\rangle$ in der Basis $|j_1 m_1\rangle |j_2 m_2\rangle$ für die zusammengesetzten Systeme $\frac{1}{2} \otimes 1$ and $1 \otimes 1$ (Das Symbol \otimes steht für die Kopplung von zwei Drehimpulsen).
- b) (15 p.) Überprüfe explizit, dass die Zerlegungen der Zustände $|\frac{5}{2}, +\frac{1}{2}\rangle$ in der Basis $|\frac{1}{2} m_1\rangle |1 m_2\rangle |1 m_3\rangle$ die man für $(\frac{1}{2} \otimes 1) \otimes 1$ und $\frac{1}{2} \otimes (1 \otimes 1)$ erhält, die gleichen sind.

Übung 2. (30 Punkte)

Nimm ein allgemeinen Spin-1/2 Zustand (also mit $|a|^2 + |b|^2 = 1$)

$$\chi = \begin{pmatrix} a \\ b \end{pmatrix}$$

- a) (10 p.) Zeige, dass es immer eine Raumrichtung $\vec{n} = (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta)$ gibt, in der χ der Eigenzustand der Spin-Komponente entlang dieser Richtung ist. $S_{\vec{n}} = \vec{n} \cdot \vec{S}$ mit Eigenwert $\hbar/2$.
- b) (15 p.) Drücke θ und ϕ durch a und b aus.
- c) (5 p.) Ist ein analoges Ergebnis auch für höhere Spin-Zustände zu erwarten?
Hinweis: Zähle die Freiheitsgrade.

Übung 3. (30 Punkte)

- a) (15 p.) Bestimme die Spin-Matrizen S_x, S_y, S_z in der Basis $|s, s_z\rangle$ für $s = 1$.
- b) (15 p.) Finde die Eigenwerte und normiere die Eigenvektoren von S_x und S_y in dieser Basis.
Hinweis: Die Beziehung $S_{\pm}|s, s_z\rangle = \hbar\sqrt{s(s+1) - s_z(s_z \pm 1)}|s, s_z \pm 1\rangle$ könnte nützlich sein.