EPR PARADOX & BELL'S INEQUALITIES

⇒ QUANTUM "INDETERMINISM"

CLASSICAL PHYSICS : POSITION & MOMENTUM OF PARTICLE ARE AT EACH TIME EXACTLY PREDICTED, ONCE WE KNOW INITIAL CONDITIONS

↕

QUANTUM PHYSICS : WE DO NOT KNOW FOR SURE IN WHICH STATE SYSTEM IS UNTIL WE MEASURE

ONLY PROBABILITIES CAN BE PREDICTED

e.g. $|\Psi\rangle$ SPIN STATE OF e^- (SPIN 1/2)

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle + |\downarrow\rangle \right)$$

$P_{\Uparrow}$ PROBABILITY THAT e^- IS FOUND IN SPIN UP STATE

$P_{\Downarrow}$ " " " " " " SPIN DOWN

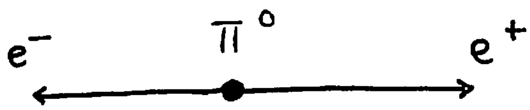
$$P_{\Uparrow} = \frac{1}{2} \qquad P_{\Downarrow} = \frac{1}{2}$$

⇒ EPR PARADOX (1935)

- EINSTEIN, PODOLSKY, ROSEN CHALLENGED QUANTUM INDETERMINISM IN A THOUGHT EXPERIMENT

  ↓

  VERSION OF D. BOHM (1952)

- 

  π^0 SPIN 0

  $e^- e^+$ PAIR ARE PRODUCED IN SPIN SINGLET STATE (ANGULAR MOMENTUM CONSERVATION)

$$|e^- e^+\rangle = \frac{1}{\sqrt{2}} \left(|\uparrow \downarrow\rangle - |\downarrow \uparrow\rangle \right)$$

SPIN STATE OF e^- SPIN STATE OF e^+

THIS IS CALLED AN ENTANGLED STATE

- ENTANGLED STATE

$$|\psi_{12}\rangle \neq |\psi_1\rangle\,|\psi_2\rangle$$

INCLUDES CORRELATIONS

PRODUCT OF 2 SINGLE PARTICLE STATES

e.g. $|\psi_{12}\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$

$|\psi_1\rangle = a_1|\uparrow\rangle + a_2|\downarrow\rangle \Rightarrow$ SPIN STATE OF PARTICLE 1

$|\psi_2\rangle = b_1|\uparrow\rangle + b_2|\downarrow\rangle \Rightarrow$ PARTICLE 2

$$|\psi_1\rangle|\psi_2\rangle = a_1 b_1|\uparrow\uparrow\rangle + a_1 b_2|\uparrow\downarrow\rangle + a_2 b_1|\downarrow\uparrow\rangle + a_2 b_2|\downarrow\downarrow\rangle$$

if $|\psi_{12}\rangle = |\psi_1\rangle|\psi_2\rangle$

$$\Downarrow$$

$$a_1 b_1 = a_2 b_2 = 0$$
$$a_1 b_2 = -a_2 b_1 = \frac{1}{\sqrt{2}}$$

$a_1 b_1 = 0 \nearrow a_1 = 0 \longrightarrow$ NOT POSSIBLE AS $a_1 b_2 = \frac{1}{\sqrt{2}}$

OR

$\searrow b_1 = 0 \longrightarrow$ NOT POSSIBLE AS $a_2 b_1 = -\frac{1}{\sqrt{2}}$

$$\therefore\ |\psi_{12}\rangle \neq |\psi_1\rangle|\psi_2\rangle$$

- EPR ARGUMENT

$|e^- e^+\rangle$ RESULTING FROM π^0 DECAY IS IN AN ENTANGLED STATE

ALICE $e^- \longleftarrow\bullet\longrightarrow e^+$ BOB

⤳ ALICE MEASURES e^- SPIN $\quad P_{e^-\Uparrow} = \frac{1}{2}$

$P_{e^-\Downarrow} = \frac{1}{2}$

⤳ BOB MEASURES e^+ SPIN $\quad P_{e^+\Uparrow} = \frac{1}{2}$

$P_{e^+\Downarrow} = \frac{1}{2}$

⤳ BUT BECAUSE $|\psi_{12}\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$

IF ALICE MEASURES $e^-\uparrow \Rightarrow$ BOB WILL WITH 100% CERTAINTY SEE $e^+\downarrow$

IF ALICE MEASURES $e^-\downarrow \Rightarrow$ BOB WILL WITH 100% CERTAINTY SEE $e^+\uparrow$

& VICE VERSA

⤳ ALICE & BOB CAN BE FAR REMOVED SUCH THAT BETWEEN TIME ALICE MADE MEASUREMENT & BOB MADE MEASUREMENT NO SIGNAL WHICH MOVES WITH SPEED OF LIGHT CAN REACH THE OTHER

$\rightsquigarrow$ EINSTEIN:

LOCALITY : NO SIGNAL CAN TRAVEL FASTER THAN SPEED OF LIGHT

QM : SEEMS TO EXHIBIT "SPOOKY ACTIONS AT A DISTANCE"

$\Downarrow$ EINSTEIN'S CONCLUSION

QM is NOT WRONG
BUT is INCOMPLETE

$|\psi\rangle$ DOES NOT REPRESENT ALL THERE IS TO KNOW ABOUT THE SYSTEM

OUR EXAMPLE :

$|\psi\rangle$ DEPENDS ONLY ON SPIN PROJ. OF PARTICLES 1 & 2

EINSTEIN : COMPLETE STATE MUST DEPEND ON SOME ADDITIONAL UNKNOWN PARAMETER λ (HIDDEN VARIABLE)

$\downarrow$

IF WE WOULD KNOW λ, WE COULD PREDICT OUTCOME OF INDIVIDUAL MEASUREMENT WITH CERTAINTY

$\therefore$ EINSTEIN : QM IS INCOMPLETE
$\hookrightarrow$ COMPLETE THEORY IS LOCAL, HIDDEN VARIABLE THEORY

⇒ BELL'S INEQUALITIES J. BELL (1964)

- EPR EXPERIMENT

EXPECTATION VALUE OF PRODUCT OF SPIN ORIENTATIONS OF e^- & e^+ IN SINGLET STATE

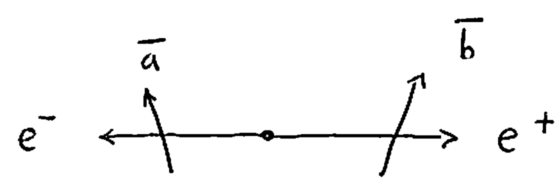
$$< \tfrac{1}{\sqrt{2}} (\uparrow\downarrow - \downarrow\uparrow) \mid \sigma_z^{(1)} \; \sigma_z^{(2)} \mid \tfrac{1}{\sqrt{2}} (\uparrow\downarrow - \downarrow\uparrow) >$$

$$= \frac{1}{2} < \uparrow\downarrow \mid \sigma_z^{(1)} \; \sigma_z^{(2)} \mid \uparrow\downarrow >$$

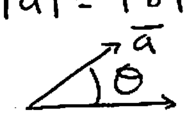
$$+ \frac{1}{2} < \downarrow\uparrow \mid \sigma_z^{(1)} \; \sigma_z^{(2)} \mid \downarrow\uparrow >$$

$$= \frac{1}{2} (1)(-1) + \frac{1}{2} (-1)(1) = \underset{\substack{\Uparrow \\ \text{PREDICTION OF QM}}}{-1}$$

- GENERALIZATION OF EPR EXP.



$|\bar{a}| = |\bar{b}| = 1$



$$\boxed{P(\bar{a}, \bar{b}) \equiv < \tfrac{1}{\sqrt{2}} (\uparrow\downarrow - \downarrow\uparrow) \mid \bar{\sigma}^{(1)} \cdot \bar{a} \;\; \bar{\sigma}^{(2)} \cdot \bar{b} \mid \tfrac{1}{\sqrt{2}} (\uparrow\downarrow - \downarrow\uparrow) >}$$

⇑

EXPECTATION VALUE OF PRODUCT OF SPIN ORIENTATIONS OF e^- ALONG DIRECTION $\bar{a}$
e^+ ALONG DIRECTION $\bar{b}$

$$P(\hat{e}_z, \hat{e}_z) = -1$$

$$\sigma_x = \frac{1}{2}(\sigma_+ + \sigma_-)$$

$$\sigma_y = \frac{1}{2i}(\sigma_+ - \sigma_-)$$

$$P(\bar{a},\bar{b}) = \frac{1}{2} \langle \uparrow\downarrow | \bar{\sigma}^{(1)}.\bar{a} \quad \bar{\sigma}^{(2)}.\bar{b} | \uparrow\downarrow \rangle$$
$$+ \frac{1}{2} \langle \downarrow\uparrow | \bar{\sigma}^{(1)}.\bar{a} \quad \bar{\sigma}^{(2)}.\bar{b} | \downarrow\uparrow \rangle$$
$$- \frac{1}{2} \langle \uparrow\downarrow | \bar{\sigma}^{(1)}.\bar{a} \quad \bar{\sigma}^{(2)}.\bar{b} | \downarrow\uparrow \rangle$$
$$- \frac{1}{2} \langle \downarrow\uparrow | \bar{\sigma}^{(1)}.\bar{a} \quad \bar{\sigma}^{(2)}.\bar{b} | \uparrow\downarrow \rangle$$

$$= \frac{1}{2} a_z b_z \langle \uparrow\downarrow | \sigma_z^{(1)} \sigma_z^{(2)} | \uparrow\downarrow \rangle$$
$$+ \frac{1}{2} a_z b_z \langle \downarrow\uparrow | \sigma_z^{(1)} \sigma_z^{(2)} | \downarrow\uparrow \rangle$$
$$- \frac{1}{2}\frac{1}{4} (a_x - i a_y)(b_x + i b_y) \langle \uparrow\downarrow | \sigma_+^{(1)} \sigma_-^{(2)} | \downarrow\uparrow \rangle$$
$$- \frac{1}{2}\frac{1}{4} (a_x + i a_y)(b_x - i b_y) \langle \downarrow\uparrow | \sigma_-^{(1)} \sigma_+^{(2)} | \uparrow\downarrow \rangle$$

$$= \frac{1}{2} a_z b_z (1)(-) + \frac{1}{2} a_z b_z (-1)(1)$$
$$- \frac{1}{8}(a_x - i a_y)(b_x + i b_y)(2)(2)$$
$$- \frac{1}{8}(a_x + i a_y)(b_x - i b_y)(2)(2)$$

$$= - a_z b_z - a_x b_x - a_y b_y = - \bar{a}.\bar{b}$$

$$\boxed{P(\bar{a},\bar{b}) = - \bar{a}.\bar{b}}$$

PREDICTION OF QM

$|\bar{a}| = |\bar{b}| = 1$

$\bar{a}.\bar{b} = \cos\theta$

- LOCAL HIDDEN VARIABLE THEORY

↳ COMPLETE KNOWLEDGE OF SYSTEM IS GIVEN BY HIDDEN VARIABLE λ
(IF WE WOULD KNOW λ, WE WOULD KNOW e.g. FOR EACH π^0 DECAY WHAT SPIN OF e^- IS)

↳ MEASURED VALUE OF e^- SPIN SEEN BY ALICE (IN UNITS $\frac{\hbar}{2}$)

$$A(\bar{a}, \lambda) = \pm 1$$

↳ MEASURED VALUE OF e^+ SPIN SEEN BY BOB

$$B(\bar{b}, \lambda) = \pm 1 \qquad (\text{IN UNITS } \tfrac{\hbar}{2})$$

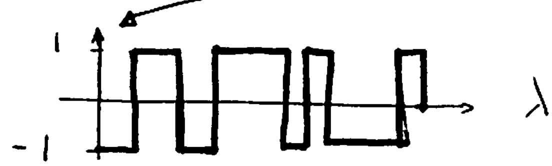
$\left.\begin{array}{l} A \text{ DOES NOT DEPEND ON } \bar{b} \\ B \text{ DOES NOT DEPEND ON } \bar{a} \end{array}\right\}$ LOCALITY ASSUMPTION (ALICE & BOB MAY BE FAR APART)

↳ $P(\lambda) > 0 \quad \forall \lambda$ PROBABILITY DISTR OF HIDDEN VARIABLE

$$\int d\lambda \; P(\lambda) = 1$$

ALICE $\langle S=0 | \bar{\sigma}^{(1)} \cdot \bar{a} | S=0 \rangle \equiv \int d\lambda \; P(\lambda) \; A(\bar{a}, \lambda)$

$= 0$
(IN SINGLET STATE $P_\Uparrow = P_\Downarrow = 1/2$)



BOB $\quad \langle S=0 | \bar{\sigma}^{(2)} \cdot \bar{b} | S=0 \rangle \equiv \int d\lambda \, P(\lambda) B(\bar{b},\lambda)$

$= 0$

↳ FOR ALIGNED DETECTORS $\bar{b} = \bar{a}$

$$A(\bar{a},\lambda) = - B(\bar{a},\lambda)$$

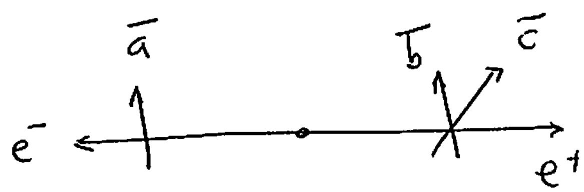
↑ MEASURED SPINS OPPOSITE

↳ $$P(\bar{a},\bar{b}) = \int d\lambda \, P(\lambda) \, A(\bar{a},\lambda) \underbrace{B(\bar{b},\lambda)}_{-A(\bar{b},\lambda)}$$

$$P(\bar{a},\bar{c}) = -\int d\lambda \, P(\lambda) \, A(\bar{a},\lambda) \, A(\bar{c},\lambda)$$

$$P(\bar{a},\bar{b}) - P(\bar{a},\bar{c}) = -\int d\lambda \, P(\lambda) \, A(\bar{a},\lambda)\left[A(\bar{b},\lambda) - A(\bar{c},\lambda)\right]$$

INSERT



↓ $A^2(\bar{b},\lambda) = 1$

SINCE $A(\bar{b},\lambda) = \pm 1$

$$P(\bar{a},\bar{b}) - P(\bar{a},\bar{c})$$

$$= -\int d\lambda \, P(\lambda) \, A(\bar{a},\lambda) \, A(\bar{b},\lambda) \left[1 - A(\bar{b},\lambda) \, A(\bar{c},\lambda)\right]$$

$$-1 \leqslant A(\bar{a},\lambda)\, A(\bar{b},\lambda) \leqslant +1$$

$$P(\lambda)\left[1 - A(\bar{b},\lambda)\, A(\bar{c},\lambda)\right] \geqslant 0$$

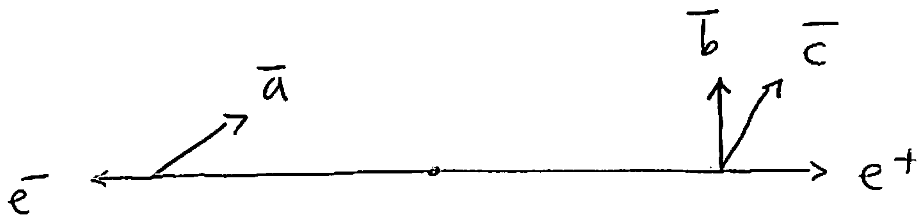
$$\left| P(\bar{a},\bar{b}) - P(\bar{a},\bar{c}) \right| \leqslant \int d\lambda \; P(\lambda)\left[1 - A(\bar{b},\lambda)\, A(\bar{c},\lambda)\right]$$

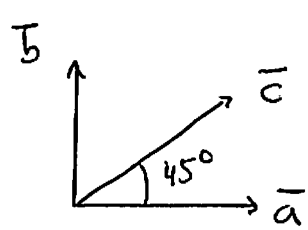
$$= 1 + P(\bar{b},\bar{c})$$

$$\boxed{\left| P(\bar{a},\bar{b}) - P(\bar{a},\bar{c}) \right| \leqslant 1 + P(\bar{b},\bar{c})}$$

BELL INEQUALITY

↳ EXAMPLE





- PREDICTION OF QM :

$$P(\bar{a},\bar{b}) = -\cos 90^\circ = 0$$

$$P(\bar{a},\bar{c}) = -\cos 45^\circ = -\frac{1}{\sqrt{2}}$$

$$P(\bar{b},\bar{c}) = -\cos 45^\circ = -\frac{1}{\sqrt{2}}$$

↓

INSERT IN BELL'S INEQUALITY

$$\underbrace{\frac{1}{\sqrt{2}}}_{0.707} \leq \underbrace{1-\frac{1}{\sqrt{2}}}_{0.293}$$

↯

∴ QM VIOLATES BELL'S INEQUALITY.

$$\boxed{\text{LOCAL HIDDEN VARIABLE THEORY } (\rightarrow \text{BELL'S INEQ.}) \text{ IS INCOMPATIBLE WITH QUANTUM MECHANICS}}$$

↓

IT CAN BE DETERMINED BY EXPERIMENT WHICH OF TWO IS CORRECT (IF ANY) !

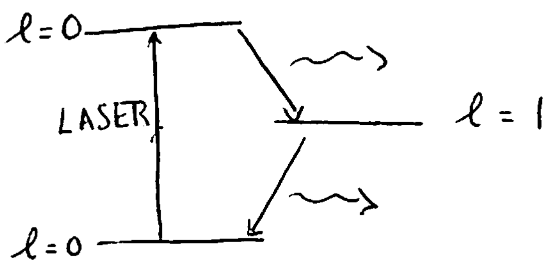
- EXPERIMENT

PIONEERING GROUP : ALAIN ASPECT *et al.*, (ORSAY, FRANCE).
PHYS. REV. LETT. 49, 91 (1982)
" " " 49, 1804 (1982)

EXP. IS DONE WITH PHOTON ENTANGLED STATES
INSTEAD OF SPIN 1/2 PARTICLES

PHOTON POLARIZATION (2 POSSIBLE STATES)
↳ TRANSVERSE ON DIRECTION OF MOTION

e^- IN CALCIUM ATOM EXCITED BY



$$|2\gamma\rangle = \frac{1}{\sqrt{2}} \left| \uparrow\uparrow + \rightarrow\rightarrow \right\rangle$$ ENTANGLED STATE

↳ LEADS TO SAME CORRELATIONS AS SPIN SINGLET STATE

∴

QM WITHSTANDS TEST BY EXPERIMENT !   LOCAL HIDDEN VARIABLE THEORY   IS DISPROVED BY EXPERIMENT