Practice Exam
Theoretical Physics 6a (QFT): WS 2017-2018
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Exercise 1. Vector + Pseudoscalar (70 points)

Consider the following Lagrangian:

$$\mathcal{L} = \underbrace{(\partial_\mu \pi^\dagger)(\partial_\mu \pi) - m_\pi^2\, \pi^\dagger \pi + \frac{1}{2} W_{\mu\nu} W^{\mu\nu} - m_\rho^2\, \rho_\mu \rho^\mu}_{\mathcal{L}_{\text{free}}} + \underbrace{i g_{\rho\pi\pi} \rho^\mu \left(\pi^\dagger \partial_\mu \pi - \pi \partial_\mu \pi^\dagger \right)}_{\mathcal{L}_{\text{I}}} \tag{1}$$

where $W_{\mu\nu} = \partial_\mu \rho_\nu - \partial_\nu \rho_\mu$ and the real field ρ^μ corresponds to the neutral vector meson ρ, where π ($\pi^\dagger$) are the pseudoscalar meson fields which absorb the charged pions $\pi^-(\pi^+)$.

(a)(15 points) Using the free Lagrangian ($\mathcal{L}_{\text{free}}$) write down the equation of motion and the Noether current for the fields, considering the global transformation $e^{-i\theta}$.

(b)(15 points) Determine the propagators of the particles in the momentum space.

(c)(20 points) For the Dyson expansion of S-Matrix at the second order (n=2) in $g_{\rho\pi\pi}$, calculate the S-matrix elements and its respective diagrams.

(d)(20 points) Using the result obtained in the previous item, calculate the total cross section for the elastic collision of $\pi^+\pi^- \to \pi^+\pi^-$.

Exercise 2. Renormalization Scalars (30 points)

Consider now the $\lambda\phi^4$ theory, with the following interaction Lagrangian

$$\mathcal{L}_I = -\frac{\lambda}{4!}\phi^4. \tag{2}$$

(a)(20 points) Using the Feynman rules and the $\overline{\text{MS}}$ scheme, calculate the one-loop diagram, for a $2 \to 2$ process for the s-channel, that is, $s \equiv (p_1 + p_2)^2 \geq 4m_\pi^2$

(b)(10 points) With this result, one can resum the lowest order diagram through a geometric series. Calculate the β-function at leading order when this sum is truncated in second order ($n = 1$). From this result of beta function what one can say about the behavior of the coupling?