Exercise sheet 9
Theoretical Physics 6a (QFT): WS 2017-2018
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Exercise 1. (50 points) : $e^+e^- \to \phi^+\phi^-$

In the lecture, we derived the differential cross section ($d\sigma/d\Omega$) for the $e^+e^- \to \mu^+\mu^-$ in the collider frame, assuming the high energy limit ($\sqrt{s} \gg m_\mu$).
For this exercise, consider the case when the muons (spin 1/2) are replaced by spinless bosons ϕ (spin 0).

(a)(20 points) Determine the possible Feynman diagrams, by calculating the second order S-matrix element (S_{fi}) for the $e^+e^- \to \phi^+\phi^-$ process. *Hint:* Notice that the scalar QED interaction needs to be considered in this case.

(b)(25 points) Calculate the unpolarized differential cross-section $d\sigma/d\Omega$ for this process in the the center of mass frame (collider frame).

(c)(05 points) Using the result obtained in the previous item, determine the total cross section.

Exercise 2. (50 points) : Compton Scattering

(a)(10 points) For the elastic collision between a photon and an electron, $\gamma + e^- \to \gamma + e^-$, express the energy of the outgoing photon in lab system (ω') as function of the electron mass (m_e), the initial photon energy (ω) and

the photon scattering angle (θ) (all in the lab system).

(b)(15 points) Calculate the second order S-matrix element (S_{fi}) for the Compton scattering off an electron and draw all the possible Feynman diagrams.

(c)(15 points) Show that the unpolarized differential cross section for the Compton scattering in the lab system can be written as

$$\left(\frac{d\sigma}{d\Omega}\right)_{\rm lab} = \frac{\alpha^2}{2m_e^2}\left(\frac{\omega'}{\omega}\right)^2\left\{\frac{\omega}{\omega'} + \frac{\omega'}{\omega} - \sin^2\theta\right\}.$$

(c)(10 points) What is the numerical value of the differential cross section (in nanobarns) when the photon energy $E_{\rm lab} = 1$ GeV and the lab scattering angle is 90°.